Essential Math for Accounting: Part II

A Review of Basic Math Essential for Accounting and Business

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Essential Math for Accounting: Part II

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BRIEF OVERVIEW

Introduction

Combined with the math review in Part I of this series, you will have a review of all the math you will need for your entire first year of accounting study … and more.

How to use this section

You do not need to read the entire math review. Simply study those topics that you feel you need to practice.

- Read the topic that you feel you need to practice.
- When you finish reading, work the "Practice" problems for that topic.
- Review the solutions and highlight the problems you missed, so you can try them again or ask your instructor or classmates for more help.

Math review in the prior volume …

This is the second part of a two-part math review. The Part I of this series contains the first math review, and includes:

- basic arithmetic operations
- rounding
- decimal operations
- percent operations
- positive and negative numbers
- how to evaluate an expression
- introduction to algebra

In this section, you will find:

Fractions: Parts of a Whole

Multiplying Fractions

Dividing Fractions

Adding Fractions

Three Ways to Find the Lowest Common Denominator

Subtracting Fractions

Ratios

Averages

The Weighted Average

Continuation of Basic Algebra Review

Essential Terminology

Equations With a Variable On Only One Side

INTRODUCTION

A continuation of previous material . . .

This Essential Math for Accounting material is a continuation of the Essential Math for Accounting in the first special report of this series. That math review covered the following topics:

- numerals
- the place-value system
- arithmetic operations
- decimal operations
- percent operations
- positive and negative numbers
- evaluating expressions
- introduction to algebra

Overview

The math review in this special report is presented with the assumption that you understand and feel reasonably comfortable with the above topics, and covers the following areas:

- explanation and use of fractions
- averages
- ratios
- continuation of basic algebra topics

Fractions: Parts of a Whole

INTRODUCTION

Place-value system and whole numbers

In the math review of the first special report in this series, we began by studying how to express numerical amounts. First we looked at whole numbers like 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, etc. These amounts are equal to or greater than 1. We expressed these amounts as place-value numerals. In the **place-value** system, values are expressed as multiples of 10, according to where numerals are placed in relation to each other.

Place-value numbers between 0 and 1

After this, we began to examine small numbers—numbers that are between 0 and 1. We then decided that we could also use the place-value system to express these small amounts, too. In the place-value system, we show these small numbers as **decimals**, such as tenths, hundredths, and so on. We use a period mark to show a decimal. For example, the amount of two-tenths can be expressed as the decimal ".2"

Example

The example below shows you the number 2,879,533.572 with each of the individual place values identified. Notice how the decimal numerals are written to the right of the decimal point.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
2,	8	7	9,	5	3	3	.	5	7	2

Percents—an alternative method

Later on, we also discovered that we could express all the same values that are shown in a place-value system in a different way—by using percents. In this approach, values are expressed as parts per 100, with 100 being an agreed-upon standard point of reference. We use the "%" symbol whenever we want to show that we are expressing values using the percent method. For example, expressing twenty-seven hundredths (.27) as a percent, we write "27%." Or, we can write 2.35 as "235%."

INTRODUCTION (continued)

Fractions—the third alternative

Fractions are simply a third alternative method for expressing exactly the same values that we can express using either the place-value system or the percent system.

Why fractions are different

A fraction is different because it expresses a value by showing some number of parts that come from a whole unit that consists of specified total number of parts. With a fraction:

- a value is expressed as some number of parts from a whole unit, and
- a whole unit can consist of any number of parts that we designate. This is very useful sometimes, and is different than the place-value system, which is based on using multiples of 10. It is also different than percents, which are based on expressions as parts per 100.

They all express the same values!

You should be clear that the place-value system, percents, and fractions are all just different ways of expressing the same values. You can always convert a value from one method to another.

So why use different methods?

Different methods are used because, in different situations, one method is sometimes easier to use or easier to understand than another method. Also, different people have different preferences. Therefore, you must be comfortable with all three methods of expressing values.

Example: alternative ways to express a value

Suppose that you wish to express the value twenty-five hundredths. You can:

- *Use decimals in the place-value numeral system:* This is .25.
- *Use percents:* In many cases, percents seem clearer to people. We can convert a decimal number to a percent, and say "twenty-five percent," written as "25%."
- *Use fractions:* A fraction would show this value as $\frac{25}{100}$, but could also use other number combinations. Next, we will study how fractions work.

INTRODUCTION (continued)

Special advantages of fractions

It is very useful to know how to use fractions because:

- many times a value will be expressed by comparing a given number of parts to the number of parts in one whole amount—such a comparison is a fraction.
- using fractions eliminates the need for rounding when calculating or expressing answers.
- it is the custom to express certain types of values as fractions.

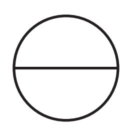
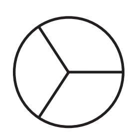
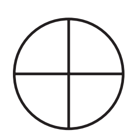
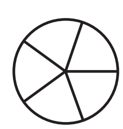
COUNTING THE PARTS IN A WHOLE AMOUNT

Overview

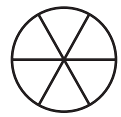
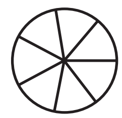
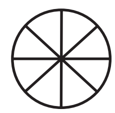
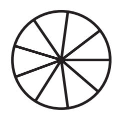
A fraction shows the number of equal-sized parts in the whole amount of something. This makes it possible for a fraction to express the whole amount of something by showing the whole as the total number of its parts.

Examples

In the following example, we are going to divide a pizza into equal parts. Therefore, the pizza is the whole amount which we will identify by its total parts.

If we divide a pizza into this many parts . . .	we could visualize this . . .	and the total parts are expressed in English as . . .
2		two-**halves**
3		three-**thirds**
4		four-**fourths**
5		five-**fifths**

COUNTING THE PARTS IN A WHOLE AMOUNT (continued)

If we divide a pizza into this many parts . . .	we could visualize this . . .	and the total parts are expressed in English as . . .
6		six-**sixths**
7		seven-**sevenths**
8		eight-**eighths**
9		nine-**ninths**

and so on . . .

We could keep dividing the pizza into smaller, equal-sized pieces. However, no matter how many parts we make, we can always express the whole amount of a pizza as the total number of its individual parts.

THE NAMES OF THE PARTS

Overview

What name to use to describe the parts in a whole amount depends upon the number of parts.

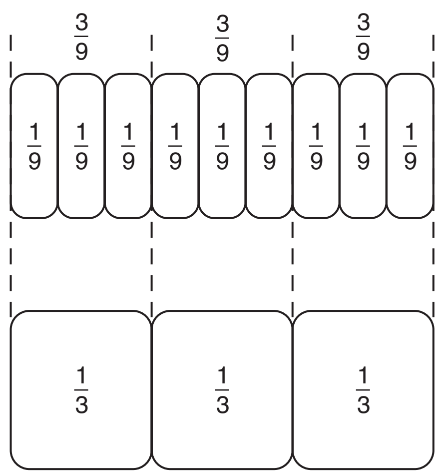
Names to use

The table on page 11 shows what names to use for some common equal-sized parts of a whole.

RAISING A FRACTION TO HIGHER TERMS (continued)

Example

Suppose that you bake a loaf of fresh bread, which you plan to serve to three guests at dinner. This means that each guest will receive one-third of the loaf. However, to your surprise, each guest brings two friends. This means that the whole loaf will now have to be divided among nine people. So, each one-third of the loaf will now have to be shared by three people. There are now more parts than before. How is each one-third to be divided now? The diagram below illustrates the new portions.



Now: 9 people share

$\frac{9}{9}$ = 1 whole loaf

Before: 3 people share

$\frac{3}{3}$ = 1 whole loaf

Because of the new situation, each $\frac{1}{3}$ has been raised to a new fraction of higher terms: $\frac{3}{9}$. However, the $\frac{3}{9}$ is still exactly equivalent to $\frac{1}{3}$ of the loaf.

Rule: raising to higher terms

To raise a fraction to higher terms, multiply *both* the numerator and the denominator of a fraction by the *same* nonzero number.

Value not changed

Multiplying both the numerator and denominator in the fraction $\frac{1}{3}$

by the same amount of 3 *does not change the value of the fraction.*

So, $\frac{1 \times 3}{3 \times 3}$ results in $\frac{3}{9}$

which is really the same total value, just expressed by *using a fraction with more parts.* Each one-third portion is now expressed as three-ninths.

RAISING A FRACTION TO HIGHER TERMS (continued)

What should I multiply by?

In the example on page 28, we multiplied the fraction by 3, because each portion had to be separated into three times more pieces. If we had wanted to separate each portion into five times more pieces, we would have multiplied the numerator and denominator by 5, and raised the fraction to 5/15. For a portion that is seven times more pieces, we would multiply by 7 and raise the fraction to 7/21, and so on.

Procedure: when you know how many times more parts

When you know the factor of how many times more total parts are needed (the denominator) or the factor of how many times more each portion (the numerator) must be, multiply the numerator and denominator by that factor.

Examples

- Raise $\frac{2}{5}$ to an equivalent fraction that has five times more total parts.

  Answer: Multiply by 5 $\frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$

- Raise $\frac{4}{7}$ to an equivalent fraction with portions of four times more parts.

  Answer: Multiply by 4 $\frac{4 \cdot 4}{7 \cdot 4} = \frac{16}{28}$

Procedure: when you only know the new numerator or denominator

Sometimes only the numerator or the denominator of the new fraction will be known to you. However, from this you can determine the correct multiple.

The table below and continued on page 30 shows how to use the numerator or the denominator to determine the correct multiple.

If …	Then …	Example
You know the numerator of the new fraction	• Divide the numerator of the old fraction into the numerator of the new fraction. • Use the result as the amount by which to multiply the old fraction.	Raise the fraction 3/4 to an equivalent fraction of higher terms which has a numerator of 15. *Answer:* • $15 \div 3 = 5$ • $\frac{3 \cdot \mathbf{5}}{4 \cdot \mathbf{5}} = \frac{15}{20}$

RAISING A FRACTION TO HIGHER TERMS (continued)

Procedure: when you only know the new numerator or denominator (continued)

If …	Then …	Example
You know the denominator of the new fraction	• Divide the denominator of the old fraction into the denominator of the new fraction. • Use the result as the amount by which to multiply the old fraction.	1. Raise the fraction 2/7 to an equivalent fraction of higher terms which has a denominator (total parts) of 84. • $84 \div 7 = 12$ • $\frac{2 \cdot \mathbf{12}}{7 \cdot \mathbf{12}} = \frac{24}{84}$ 2. Raise the fraction 2/7 to an equivalent fraction of twenty-eighths. • $28 \div 7 = 4$ • $\frac{2 \cdot \mathbf{4}}{7 \cdot \mathbf{4}} = \frac{4}{28}$

More examples

• $\frac{2}{3} = \frac{?}{6}$ *Answer:* $\frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6}$	• $\frac{4}{9} = \frac{?}{45}$ *Answer:* $\frac{4 \cdot 5}{9 \cdot 5} = \frac{20}{45}$
• $\frac{1}{5} = \frac{8}{?}$ *Answer:* $\frac{1 \cdot 8}{5 \cdot 8} = \frac{8}{40}$	• $\frac{3}{4} = \frac{9}{?}$ *Answer:* $\frac{3 \cdot 3}{4 \cdot 3} = \frac{9}{12}$

PRACTICE

REINFORCEMENT PROBLEMS: RAISING FRACTIONS TO HIGHER TERMS

Instructions: Based on the information given to you about the new fraction, raise the old fraction to higher terms. Write your answer in the space provided.

Fraction	New Fraction	Answer	Fraction	New Fraction	Answer
1. 2/6	has 4 times more total parts		11. 2/ 5	parts are hundredths	
2. 2/6	parts are twenty-fourths		12. 2/5	parts are thousandths	
3. 5/8	15/?		13. 2/5	parts are eightieths	
4. 19/13	?/65		14. 23/36	?/108	
5. 10/12	numerator is 3 times greater		15. 2/11	has 7 times more total parts	
6. 12/9	parts are thirty-sixths		16. 21/15	?/45	
7. 2/9	?/27		17. $x/7$	parts are twenty-firsts	
8. 14/43	?/86		18. $15/2x$	$?/18x$	
9. 5/8	40/?		19. x/y	$4x/?$	
10. 7/11	each portion has 5 times more parts in it		20. $(x - 1)/y$	$?/3y$	

SOLUTIONS

Fraction	New Fraction	Answer	Fraction	New Fraction	Answer
1. 2/6	has 4 times more total parts	8/24	11. 2/ 5	parts are hundredths	40/100
2. 2/6	parts are twenty-fourths	8/24	12. 2/5	parts are thousandths	400/1,000
3. 5/8	15/?	15/24	13. 2/5	parts are eightieths	32/80
4. 19/13	?/65	95/65	14. 23/36	?/108	69/108
5. 10/12	numerator is 3 times greater	30/36	15. 2/11	has 7 times more total parts	14/77
6. 12/9	parts are thirty-sixths	48/36	16. 21/15	?/45	63/45
7. 2/9	?/27	6/27	17. $x/7$	parts are twenty-firsts	$3x/21$
8. 14/43	?/86	28/86	18. $15/2x$	$?/18x$	$135/18x$
9. 5/8	40/?	40/64	19. x/y	$4x/?$	$4x/4y$
10. 7/11	each portion has 5 times more parts in it	35/55	20. $(x - 1)/y$	$?/3y$	$3(x - 1)/3y$

LOWER AND LOWEST TERMS

Overview

Just as it is sometimes necessary to raise a fraction to higher terms, it is also sometimes necessary to reduce a fraction to an equivalent fraction of lower terms. Normally a fraction is reduced to lower terms in order to express the fraction more clearly. The clearest way to express a fraction is by converting it into *lowest* terms.

Definition: lower terms

Reducing a fraction to lower terms means converting a fraction into an equivalent faction which has fewer parts.

Examples

Fraction	Lower term equivalent fraction
24/36	12/18
150/240	15/24
120/200	60/100

Definition: lowest terms

Reducing a fraction to lowest terms means finding an equivalent fraction which:

- has the smallest possible whole number denominator, and
- the numerator and denominator cannot be evenly divided by the same number, except 1.

Examples

Fraction	Lowest term equivalent fraction
24/36	2/3
150/240	5/8
120/200	3/5

HOW TO REDUCE TO LOWER TERMS

Method

To reduce a fraction to lower terms, divide both the numerator and denominator by the same whole number, except 1, that leaves no remainder.

HOW TO REDUCE TO LOWER TERMS (continued)

Example

- Reduce $\frac{20}{30}$ to lower terms: $\frac{20 \div 5}{30 \div 5} = \frac{4}{6}$

HOW TO REDUCE TO LOWEST TERMS

Overview

Generally, it is more important to be able to reduce a fraction to the lowest terms possible. Fractional answers to problems are usually shown in lowest terms. This always results in the most easy-to-understand fraction.

Trial and error method

The table below shows the steps to use in the trial-and-error method. (The example reduces the fraction 120/200 to its lowest terms of 3/5.)

Step	Action	Example
1	Examine the numerator and the denominator until you find any whole number that will divide evenly into both of them.	Reduce the fraction $\frac{120}{200}$ to lowest terms: • Both 120 and 200 can be evenly divided by 10.
2	Divide both the numerator and denominator by that whole number, and use the quotients as as a new fraction.	$\frac{120 \div 10}{200 \div 10} = \frac{12}{20}$
3	Repeat **STEPS 1** and **2** as many times as necessary until no more whole numbers will divide evenly into the numerator and denominator, except 1.	$\frac{12 \div 4}{20 \div 4} = \frac{3}{5}$ • Both 12 and 20 can be evenly divided by 4. The fraction cannot be reduced any lower.

Greatest common divisor (GCD)

The **greatest common divisor** is a number that will immediately reduce a fraction to its lowest terms, without needing to use trial and error.

HOW TO REDUCE TO LOWEST TERMS (continued)

GCD method for reducing fractions

1. Find the greatest common divisor.
2. Divide it into both the numerator and denominator.
3. Use the quotients as the new fraction.

How to find a GCD

The table below shows the steps to find a greatest common divisor. (The example finds the greatest common divisor for 120/200.)

Step	Action	Example
1	Examine the numerator and denominator. Divide the smaller number into the larger number.	To find the GCD of $\frac{120}{200}$, divide: $120\overline{)200}$ = 1, remainder of **80**
2	If there is a remainder, divide it into the divisor.	$80\overline{)120}$ = 1, remainder of **40**
3	Continue dividing each new remainder into the divisor until you obtain a division for which there is no remainder.	$40\overline{)80}$ = 2, no remainder
4	The final divisor (here, 40) is the greatest common divisor.	$\frac{120 \div 40}{200 \div 40} = \frac{3}{5}$

Not every fraction can be reduced

Some fractions, such as 37/39, have a greatest common divisor of only 1. This means that the fraction is already in the lowest possible terms. Sometimes this is obvious, but frequently you will have to go through the steps to find out.

PRACTICE

REINFORCEMENT PROBLEMS: REDUCING FRACTIONS TO LOWEST TERMS

1. *Instructions:* Reduce each of the fractions shown below to its lowest terms. Write your answer in the space provided next to each fraction. **Use both methods**.

Fraction	Lowest Terms	Fraction	Lowest Terms
a. 12/60		h. 70/130	
b. 24/40		i. 9/12	
c. 14/112		j. 54/126	
d. 42/84		k. 78/96	
e. 91/156		l. 10/35	
f. 5/70		m. 6/27	
g. 85/306		n. 204/210	

2. *Review:* What is the difference between raising a fraction to higher terms and reducing a fraction to lowest terms? Why are these procedures done?

SOLUTIONS

1.

Fraction	Lowest Terms	Fraction	Lowest Terms
a. 12/60	1/5	h. 70/130	7/13
b. 24/40	3/5	i. 9/12	3/4
c. 14/112	1/8	j. 54/126	3/7
d. 42/84	1/2	k. 78/96	13/16
e. 91/156	7/12	l. 10/35	2/7
f. 5/70	1/14	m. 6/27	2/9
g. 85/306	5/18	n. 204/210	34/35

2. Each procedure is done in order to express a fraction so it is either easier to use in a calculation or easier to understand as an answer. This will depend on each situation. Answers that are fractions are almost always reduced to lowest terms, because smaller fractions are easier to understand.

Raising a fraction to higher terms means multiplying both the numerator and denominator by the same number, so both the numerator and denominator in the new fraction are bigger. However, the resulting fraction is *still equivalent in value to the original.*

Reducing a fraction to lowest terms means dividing the numerator and denominator by the same number, so both the numerator and denominator are reduced to the point where each can be evenly divided only by 1. However, the resulting fraction is *still equivalent in value to the original*.

CONVERTING FRACTIONS INTO DECIMALS AND INTO PERCENT

Overview

The need to convert a fraction into a decimal or a percent form is a common occurrence. This is especially true for numbers that are less than one.

Although all three forms (decimal, percent, and fraction) can express exactly the same value, sometimes it is more convenient to work with numbers in one form than another. This situation may arise because of calculations you are doing, or it may simply involve writing a report when you know that the readers prefer and expect to see numbers expressed in a certain way.

Procedure

The table below shows how to convert fractions into decimals and percents.

Step	Action	Example
1	Select a fraction and decide how many places to the right of the decimal point you want to express it.	Convert 3/8 to a decimal, expressing the answer to the thousandths place (3 places to the right of the decimal point). Then express the number as a percent.
2	Divide the denominator into the numerator, until the quotient is *one place more* than the required number of places.	.3750 8 | 3.0000 24 60 56 40 40 0 Quotient to four places
3	If necessary, round the decimal to the desired place, using the rules for rounding. (See Volume 1.)	In this case, you do not need to apply the rules for rounding. .3750 is simply .375
4	To convert the decimal to a percent, multiply by 100, and write the "%" symbol.	$.375 \times 100 = 37.5\%$

Same value

Notice in the example above that 3/8, .375, and 37.5% all express exactly the same value. Only the form of expression is different.

CONVERTING FRACTIONS INTO DECIMALS AND INTO PERCENT (continued)

Nonterminating quotients

A **nonterminating quotient** is a quotient that never ends. You can continue dividing, and there will always be a remainder. These kinds of answers always need to be rounded to a specified number of places. The table below shows you an example.

Step	Action	Example
1	Select a fraction and decide how many places to the right of the decimal point you want to express it.	Convert 3/7 to a decimal. Your instructor requires decimal accuracy to thousandths place.
2	Divide the denominator into the numerator, until the quotient is *one place more* than the required number of places. *Note:* Notice that the quotient is nonterminating.	.4285 ← Quotient to four places 7) 3.0000 28 20 14 60 56 40 35
3	Round the decimal to the desired place, using the rules for rounding. (See Volume 1.)	.4285 rounded is .429
4	To convert the decimal to a percent, multiply by 100, and write the "%" symbol.	The problem did not ask for conversion to a percent. However, the percent is 42.9%.

What if I have a mixed numeral?

If you have a mixed numeral, convert it to a fraction and then follow the procedure above. The method for converting a mixed numeral to a fraction is on page 23.