Essential Math for Accounting

BRIEF OVERVIEW

Introduction

This review is designed to give you a review of all the math you will need for this book and for beginning your work in any introductory accounting text. Combined with the math review in the next book of this series (Volume 2), you will have a review of all the math you will need for your entire first year of accounting study … and more.

How to use this section

You do not need to read the entire math review. Simply study those topics that you feel you need to practice.

- Read the topic that you feel you need to practice.
- When you finish reading, work the "Practice" problems for that topic.
- Review the solutions and highlight the problems you missed, so you can try them again or ask your instructor or classmates for more help.

Math review in the next volume …

This review is the first in a series of two. The disk in the second book (Volume 2) continues the math review, which includes:

- explanation and use of fractions
- continuation of basic algebra topics

In this section, you will find:

General Introduction: Numerals are Symbols for Amounts

The Place-Value Numeral System

Arithmetic Operations

Decimals

Percent

Positive and Negative Numbers

Introduction to Algebra and Equations

▼ General Introduction: Numerals are Symbols for Amounts

NUMERALS

Importance of numerals

We all see and use numerals every day, usually without giving them much thought. However, the individual numerals are really a foundation of all of our mathematics! Let's take a moment to be sure that we are comfortable about what numerals really mean and how they function.

Numerals are symbols

A numeral is a symbol for an amount of something. In the number system with which most of us are familiar, there are ten symbols. For example, suppose that some of us are hungry. I suggest that we order pizza. How many pizzas? We can use symbols to express the number of pizzas we want:

Numeral Symbol	Written as …	Number of Pizzas
0	"zero"	
1	"one"	●
2	"two"	● ●
3	"three"	● ● ●
4	"four"	● ● ● ●
5	"five"	● ● ● ● ●
6	"six"	● ● ● ● ● ●
7	"seven"	● ● ● ● ● ● ●
8	"eight"	● ● ● ● ● ● ● ●
9	"nine"	● ● ● ● ● ● ● ● ●

You can see that there is a numeral symbol for each individual amount from zero units to nine units or, in this case, pizzas.

NUMERALS (continued)

Only ten symbols are used

Suppose that we wanted to order ten pizzas, or eleven, or twelve … or twenty. Could we invent more symbols for more units? Yes, that would be possible. For example, we could create a symbol like "*" for the number ten, or perhaps the symbol "Y" for the number eleven, and so on; however, this is not practical. We would need an unimaginably large number of symbols for all the possible numbers, and we could never remember them! For this reason, our number system uses only ten symbols, but in a very clever way.

▼ *The Place-Value Numeral System*

WHAT THE SYSTEM IS

Introduction

So far, we have only been able to express amounts up to nine units of something. Clearly, this is not enough. But if we are limited to only ten symbols, what can we do? A long, long time ago, the clever people who invented our number system gave this problem a great deal of thought. Their solution was something called the "place-value" numeral system.

Definition: the place-value numeral system

The place-value numeral system is a method that determines the value of a numeral by how it is positioned relative to other numerals.

Numeral groups

In the place-value numeral system, the basic ten symbols are placed into groups. The position—or place—of each group determines the size of the numerals within the group. Below is a chart of some number groups as they are positioned in a place-value system.

bigger ←			
Thousands	Hundreds	Tens	Ones

READING AND WRITING NUMERALS IN EACH PLACE

The "ones" place

The first, and smallest, place begins on the right. This is called the "ones" place because all the numerals are counted in units of one.

Examples

Below are examples with the numeral "three" and the numeral "seven," which are three units of one and seven units of one.

The numeral "three" is placed like this:			
Thousands	Hundreds	Tens	Ones
			3

The numeral "seven" is placed like this:			
Thousands	Hundreds	Tens	Ones
			7

The "tens" place

The next place is the "tens" place. All numerals written in this position are counted in *units of ten*, rather than units of one as in the "ones" place. For example, in the "tens" place, a "2" means two units of ten; in other words, twenty. A "3" in the "tens" place means three units of ten, which is thirty, and so on. Each greater numeral in the "tens" place adds another unit of ten.

Examples

These examples show you how to write and say some numerals in the "tens" place.

The number "ten"			
Thousands	Hundreds	Tens	Ones
		1	0

Saying the number:
- One unit of ten is "ten"
- No units of one is silent

The number is called "ten"

The number "twelve"			
Thousands	Hundreds	Tens	Ones
		1	2

Saying the number:
- One unit of ten is "ten"
- Two units of one is "two"

The number is called "twelve"

READING AND WRITING NUMERALS IN EACH PLACE (continued)

The number "twenty-two"			
Thousands	Hundreds	Tens	Ones
		2	**2**

Saying the number:
- Two units of ten is "twenty"
- Two units of one is "two"

The number is called "twenty-two"

The number "thirty-five"			
Thousands	Hundreds	Tens	Ones
		3	**5**

Saying the number:
- Three units of ten is "thirty"
- Five units of one is "five"

The number is called "thirty-five"

The number "forty-eight"			
Thousands	Hundreds	Tens	Ones
		4	**8**

Saying the number:
- Four units of ten is "forty"
- Eight units of one is "eight"

The number is called "forty-eight"

The number "ninety-nine"			
Thousands	Hundreds	Tens	Ones
		9	**9**

Saying the number:
- Nine units of ten is "ninety"
- Nine units of one is "nine"

The number is called "ninety-nine"

Note: All the numbers from twenty-one to ninety-nine are always hyphenated whenever they are spelled.

How about fourteen pizzas? So are we ready to order our pizzas now? Suppose that we have decided to order fourteen pizzas. The place-value number system neatly allows us to put a "1" in the "tens" place to signify ten, and then put a "4" in the "ones" place to signify four. Presto! We have easily expressed a total of **fourteen** pizzas!

"Digits"

When a number is represented by several numerals, each individual numeral is often called a "digit." For example, we represent the number eighty-three by using the numerals "8" and "3" like this: 83. The numeral "8" is one digit and the numeral "3" is another digit.

READING AND WRITING NUMERALS IN EACH PLACE (continued)

The "hundreds" place

The next place is the "hundreds" place. All numbers written in this position are counted in *units of one hundred*. For example, in the "hundreds" place, a "2" means two units of one hundred; in other words, two hundred. A "3" in the "hundreds" place means three units of one hundred, which is three hundred, and so on. In the "hundreds" place, each greater numeral adds another unit of one hundred.

Examples

Below are examples of some numbers in the "hundreds" place.

The number "one hundred thirty-two"

Thousands	Hundreds	Tens	Ones
	1	3	2

Saying the number:

- One unit of one hundred is "one hundred"
- Three units of ten is "thirty"
- Two units of one is "two"

The number is called "one hundred thirty-two"

The number "two hundred fifty"

Thousands	Hundreds	Tens	Ones
	2	5	0

Saying the number:

- Two units of one hundred is "two hundred"
- Five units of ten is "fifty"
- Zero units of one is silent

The number is called "two hundred fifty"

The number "three hundred four"

Thousands	Hundreds	Tens	Ones
	3	0	4

Saying the number:

- Three units of one hundred is "three hundred"
- Zero units of ten is silent
- Four units of one is "four"

The number is called "three hundred four"

Note: The word "and" is not used.

The number "eight hundred ninety-nine"

Thousands	Hundreds	Tens	Ones
	8	9	9

Saying the number:

- Eight units of one hundred is "eight hundred"
- Nine units of ten is "ninety"
- Nine units of one is "nine"

The number is called "eight hundred ninety-nine"

READING AND WRITING NUMERALS IN EACH PLACE (continued)

The "thousands" place

The next place is the "thousands" place. All numbers written in this position represent units of one thousand. For example, in the "thousands" place, a "2" means two units of one thousand; in other words, two thousand. A "3" in the "thousands" place means three units of one thousand, which is three thousand, and so on. Each greater numeral adds one thousand.

Examples

Below are examples of some numbers in the "thousands" place.

The number "one thousand seven hundred fifty-six"			
Thousands	**Hundreds**	**Tens**	**Ones**
1,	**7**	**5**	**6**

Saying the number:

- One unit of one thousand is "one thousand"
- Seven units of one hundred is "seven hundred"
- Five units of ten is "fifty"
- Six units of one is "six"

The number is called "one thousand seven hundred fifty-six"

The number "two thousand thirty"			
Thousands	**Hundreds**	**Tens**	**Ones**
2,	**0**	**3**	**0**

Saying the number:

- Two units of one thousand is "two thousand"
- Zero units of one hundred is silent
- Three units of ten is "thirty"
- Zero units of one is silent

The number is called "two thousand thirty"

Using commas

When numbers become larger, it is more difficult to quickly identify the number places. To make it easier to see the number places, a comma is entered after every third number place, beginning from the right.

Examples:

- The number "one thousand seven hundred fifty-six" is written "1,756."
- The number "two thousand thirty" is written "2,030."

READING AND WRITING NUMERALS IN EACH PLACE (continued)

Places are multiples of ten

Notice that each greater place in the place-value table is a multiple of ten times the preceding place value. This is sometimes called a "base ten" system.

- "10" is ten ones
- "100" is ten tens
- "1,000" is ten hundreds

… and so on, with each new place being ten times the preceding one.

A bigger place-value table

You can see that each time we have looked at a new place of numbers, the size of the numbers at the new place in the chart is ten times greater than the numbers in the previous place. Below is a place-value chart that shows place-value numerals up to one billion. Of course, you could continue to increase the chart to ten billion, one hundred billion, one trillion, and so on.

Billions	Hundred Millions	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
7,	5	7	4,	3	0	8,	2	9	1

The amount you see above is written as "7,574,308,291." It is called "seven billion, five hundred seventy-four million, three hundred eight thousand, two hundred ninety-one."

PRACTICE

SOLUTIONS FOR THE PLACE-VALUE SYSTEM BEGIN ON PAGE 12.

REINFORCEMENT PROBLEMS: THE PLACE-VALUE SYSTEM

1. **Write the words in numbers.** For each amount on the left that is expressed in words, express that same amount using numbers. The first item is an example.

Amount written in words	Amount written in numbers
a. Three thousand four hundred eighty-eight	3,488
b. Eighty-nine	
c. Two hundred twelve	
d. Nine hundred seven	
e. Twenty-two thousand, six hundred seventy-two	
f. Eleven thousand three	
g. One hundred fifty-four thousand, seven hundred thirty-three	
h. Two million, sixty-five thousand, ninety-one	
i. Ten million, two hundred eighty-six thousand, four hundred thirty-three	
j. Thirty-nine thousand, one hundred thirty-nine	
k. Seventeen thousand, four hundred twenty-seven	
l. Five thousand, seven hundred thirty-five	

2. **Write the numbers in words.** For each amount, express the number using words.

Amount written in numbers	Amount written in words
a. 257	
b. 33	
c. 4,079	
d. 7,294	
e. 12,370	
f. 97	
g. 909	
h. 54	
i. 47,882	

3. **Identify the place-value.** For each number in bold type, identify what place value place it is part of. The first two items are examples.

Number	Place-Value
2,**4**22	hundreds
3**5**,077	
846	
2**3**9,555	
7,**3**97,137	
5,**0**02	
1,**8**72	

Number	Place-Value
19**8**	ones
28,0**0**9	
11,305	
4,580,423	
9,**2**22	
812,3**3**1	
17,99**9**	

SOLUTIONS

PRACTICE QUESTIONS FOR THE PLACE-VALUE SYSTEM BEGIN ON PAGE 11.

REINFORCEMENT PROBLEMS: THE PLACE-VALUE SYSTEM

1.

Amount written in words	Amount written in numbers
a. Three thousand four hundred eighty-eight	3,488
b. Eighty-nine	89
c. Two hundred twelve	212
d. Nine hundred seven	907
e. Twenty-two thousand, six hundred seventy-two	22,672
f. Eleven thousand three	11,003
g. One hundred fifty-four thousand, seven hundred thirty-three	154,733
h. Two million, sixty-five thousand, ninety-one	2,065,091
i. Ten million, two hundred eighty-six thousand, four hundred thirty-three	10,286,433
j. Thirty-nine thousand, one hundred thirty-nine	39,139
k. Seventeen thousand, four hundred twenty-seven	17,427
l. Five thousand, seven hundred thirty-five	5,735

2.

Amount written in numbers	Amount written in words
a. 257	two hundred fifty-seven
b. 33	thirty-three
c. 4,079	four thousand seventy-nine
d. 7,294	seven thousand two hundred ninety-four
e. 12,370	twelve thousand three hundred seventy
f. 97	ninety-seven
g. 909	nine hundred nine
h. 54	fifty-four
i. 47,882	forty-seven thousand eight hundred eighty-two

3.

Number	Place-Value
2,**4**22	hundreds
3**5**,077	thousands
846	hundreds
2**3**9,555	ten thousands
7,**3**97,137	hundred thousands
5,**0**02	hundreds
1,**8**72	hundreds

Number	Place-Value
19**8**	ones
28,0**0**9	tens
11,305	ten thousands
4,580,423	millions
9,**2**22	hundreds
812,3**3**1	tens
17,99**9**	ones

Arithmetic Operations

HOW TO ADD NUMBERS

Overview

Addition is the most common of all the arithmetic operations that you will do. The essential idea to remember is that when you are doing addition, you simply start at the top of a place-value column, and add the digits in that column. It really does not matter how big the numbers are that you are adding, because you are only adding one column at a time.

Symbol for addition

To signify that numbers are being added, the "+" symbol is used. If numbers are shown vertically, the symbol is placed to the left of the last number. If numbers are shown horizontally, the symbol is placed between the numbers.

Vertical:

$$\begin{array}{r} 15 \\ 12 \\ +\ \underline{20} \end{array}$$

Horizontal: 15 + 12 + 20

Procedure

The following table shows you the steps for addition. Suppose that you want to add these numbers: 31, 5, 17, and 178.

Step	Action	Example
1	**Align the numbers** so that the individual digits in each place-value form a column. Draw a line under the bottom number.	$\begin{array}{rrrr} & & 3 & 1 \\ & & & 5 \\ & & 1 & 7 \\ + & 1 & 7 & 8 \\ \hline \end{array}$
2	Beginning with the lowest place-value column that is not totaled, **add the numbers in a column**. Add from the top down. *Note:* It will be easiest if you develop the habit of keeping a mental "running total" as you add each number in the column. These totals are shown by the small numbers. Do not write these down. You would say to yourself: "1 plus 5 is 6." Then, "6 plus 7 is 13" and so on until the end of the column.	$\begin{array}{rrrrl} & & 3 & 1 & \\ & & & 5 & _{6} \\ & & 1 & 7 & _{13} \\ + & 1 & 7 & 8 & _{21} \\ \hline \end{array}$

(table continued on next page)

HOW TO ADD NUMBERS (continued)

Step	Action	Example
3	**Write the total** of the column (called the **sum**) at the bottom of the column. *Rule:* If the sum exceeds 9, put the digits of the sum into the columns to which they correspond. For example, the total of the "ones" column is 21. Here, we want to show this as one unit of "ones" and two units of "tens." Write the two units of ten at the top of the "tens" column, where it will later be added with the other "tens." The 1 remains in the "ones" unit column.	$\begin{array}{rrr} & {}^{2} & \\ & 3 & 1 \\ & & 5 \\ & 1 & 7 \\ +\;1 & 7 & 8 \\ \hline & & 1 \end{array}$
4	If there are more place-value columns to add, **return to Step 2, and begin the next column**. **Adding the "tens" column:**	$\begin{array}{rrr} {}^{1} & {}^{\mathbf{2}} & \\ & \mathbf{3}_{5} & 1 \\ & & 5 \\ & \mathbf{1}_{6} & 7 \\ +\;1 & \mathbf{7}_{13} & 8 \\ \hline & \mathbf{3} & 1 \end{array}$
	Adding the "hundreds" column: There are no more columns to add, so you are finished.	$\begin{array}{rrr} {}^{\mathbf{1}} & {}^{2} & \\ & 3 & 1 \\ & & 5 \\ & 1 & 7 \\ +\;\mathbf{1}_{2} & 7 & 8 \\ \hline 2 & 3 & 1 \end{array}$

Checking your work

It is quite easy to make a mistake when adding a column of numbers, even if you are using a calculator. To check your totals, repeat the process, but add from the bottom to the top. Working in reverse order minimizes your chances of making the same mistake twice.

HOW TO ADD NUMBERS (continued)

Example with more numbers

Add the following five numbers: 2,196 + 13,099 + 350 + 12,454 + 41.

Step 1: Align the numbers

```
     2,  1  9  6
  1  3,  0  9  9
         3  5  0
  1  2,  4  5  4
+           4  1
----------------
```

Steps 2 and 3: Add the “ones” place-value column. 20 is zero units of one and two units of ten. Write two units of ten into the “tens” column.

```
            2
     2,  1  9  6
  1  3,  0  9  9  15
         3  5  0  15
  1  2,  4  5  4  19
+           4  1  20
----------------
               0
```

Step 4: Add the “tens” column. Thirty-four tens is really four units of ten and three units of one hundred. Write the three units of one hundred into the “hundreds” column.

```
         3  2
     2,  1  9 11  6
  1  3,  0  9 20  9
         3  5 25  0
  1  2,  4  5 30  4
+           4 34  1
-------------------
            4     0
```

Step 4: Add the “hundreds” column. Eleven hundreds is really one unit of one hundred and one unit of one thousand. Write the one unit of one thousand into the “thousands” column.

```
     1   3     2
     2,  1  4  9  6
  1  3,  0  4  9  9
         3  7  5  0
  1  2,  4 11  5  4
+              4  1
-------------------
         1     4  0
```

Step 4: Add the “thousands” column. Eight units of one thousand does not exceed nine, so no units need to be carried to the “ten thousands” column.

```
     1     3  2
     2, 3  1  9  6
  1  3, 6  0  9  9
           3  5  0
  1  2, 8  4  5  4
+             4  1
------------------
     8,    1  4  0
```

Step 4: Add the “ten thousands” column. There are two units of ten thousand, so this is twenty thousand. There are no more columns to add. The sum is 28,140.

```
      1  3  2
     2,  1  9  6
  1  3,  0  9  9
         3  5  0
 1_2 2,  4  5  4
+           4  1
----------------
  2  8,  1  4  0
```

Characteristics of addition

Numbers can be added in any order

- Adding 5 + 12 + 8 is a sum of 25
- Adding 12 + 5 + 8 is also a sum of 25

Numbers can be added in any groups

- Adding (5 + 12) + 8 or 5 + (12 + 8) both give the same answer of 25

PRACTICE

PRACTICE

REINFORCEMENT PROBLEM: ADDITION

1. Add the numbers shown in the left column of the table and show your answers in the right column.

Add the following numbers …	Answer
a. 4; 7; 9; 3	
b. 25; 189; 101; 56; 7	
c. 111; 390; 459; 285	
d. 1,003; 2,597; 144	
e. 2,091; 3,511; 9,972	
f. 1,461; 5,238; 9,499	
g. 19,884; 52,040; 27,318	
h. 299,351; 327,500	
i. 1,345,739; 12,905,476	
j. 11,005; 434; 899; 21,927	
k. 450; 50; 300	
l. 200; 1,000; 300	

SOLUTIONS

REINFORCEMENT PROBLEM: ADDITION

1.

Add the following numbers …	Answer
a. 4; 7; 9; 3	23
b. 25; 189; 101; 56; 7	378
c. 111; 390; 459; 285	1,245
d. 1,003; 2,597; 144	3,744
e. 2,091; 3,511; 9,972	15,574
f. 1,461; 5,238; 9,499	16,198
g. 19,884; 52,040; 27,318	99,242
h. 299,351; 327,500	626,851
i. 1,345,739; 12,905,476	14,251,215
j. 11,005; 434; 899; 21,927	34,265
k. 450; 50; 300	800
l. 200; 1,000; 300	1,500

HOW TO SUBTRACT NUMBERS

Overview

Subtraction is the process of finding the difference between two numbers. The difference between two numbers simply means the amount by which one number is greater than another number.

Example

If we subtract 7 from 10, we find that the difference is 3. This means that 10 is greater than 7 by the amount of 3.

Symbol for subtraction

To signify that numbers are being subtracted, the "–" symbol is used. If numbers are shown vertically, the symbol is placed to left of the bottom number. If numbers are shown horizontally, the symbol is placed between the numbers.

Vertical: $\begin{array}{r} 50 \\ -\underline{20} \end{array}$ Horizontal: 50 – 20

Procedure

The following table shows you the steps for subtraction. Suppose that you are planning a picnic. You have 195 paper plates and you will need 382. How many more do you need? (You need to find the difference between the numbers.)

<table>
<tr><th>Step</th><th colspan="2">Action</th><th>Example</th></tr>
<tr><td>1</td><td colspan="2">Align the numbers so that the individual digits are in the correct place-value columns. When aligning the numbers, put the larger number on top.</td><td>$\begin{array}{rrrr} & 3 & 8 & 2 \\ - & 1 & 9 & 5 \\ \hline \end{array}$</td></tr>
<tr><td rowspan="4">2</td><td colspan="2">Go to the lowest place-value column that has not been subtracted.</td><td>$\begin{array}{rrrr} & & \scriptstyle 7 & \scriptstyle 12 \\ & 3 & \cancel{8} & \cancel{2} \\ - & 1 & 9 & 5 \\ \hline \end{array}$</td></tr>
<tr><th>If …</th><th colspan="2">Then …</th></tr>
<tr><td>the bottom number in the column is smaller than the top number …</td><td colspan="2">… go to Step 3.</td></tr>
<tr><td>the bottom number in the column is bigger than the top number (here, the 5 is bigger than the 2) …</td><td colspan="2">… increase the top number in that column by borrowing a unit from the column next to it.

One unit borrowed from the "tens" column adds ten to the "ones" column. So, the "ones" column increases to 12 (10 plus the 2). The 8 in the "tens" column is reduced to 7. Cross out the old numbers and write in the changed numbers.</td></tr>
</table>

(table continued on next page)

HOW TO SUBTRACT NUMBERS (continued)

Step	Action	Example
3	**Subtract the bottom number in the column from the top number**, and write the difference under the line.	$\begin{array}{rrrr} & & 7 & 12 \\ & 3 & \cancel{8} & \cancel{2} \\ - & 1 & 9 & 5 \\ \hline & & & 7 \end{array}$
4	If there are more columns to subtract, return to Step 2 and repeat the procedure with the other columns: **Subtracting the "tens" column:** Because the 9 is greater than the 7, we need to borrow one unit from the "hundreds" column. One unit from the "hundreds" column adds 10 to the "tens" column, so the 7 is increased to 17. The difference between 9 and 17 is 8.	$\begin{array}{rrrr} & & 17 & \\ & 2 & \cancel{7} & 12 \\ & \cancel{3} & \cancel{8} & \cancel{2} \\ - & 1 & 9 & 5 \\ \hline & & 8 & 7 \end{array}$
	Subtracting the "hundreds" column: 1 is less than 2, and the difference is one. You need 187 more paper plates.	$\begin{array}{rrrr} & & 17 & \\ & 2 & \cancel{7} & 12 \\ & \cancel{3} & \cancel{8} & \cancel{2} \\ - & 1 & 9 & 5 \\ \hline & 1 & 8 & 7 \end{array}$

Example without steps

Here is an example of subtraction without listing the steps:

Subtract 4,301 from 9,037:

$$\begin{array}{rrrrr} & 8 & 10 & & \\ & \cancel{9,} & \cancel{0} & 3 & 7 \\ - & 4, & 3 & 0 & 1 \\ \hline & 4, & 7 & 3 & 6 \end{array}$$

The difference is 4,736.

Example: moving two places to borrow

Occasionally, you may have to perform a subtraction operation with a number that makes it necessary to move two or more places to the left to find a number from which to borrow.

Example: Subtract 897 from 5,001.

HOW TO SUBTRACT NUMBERS (continued)

Step	Action	Example
1	**Align the numbers**.	$\begin{array}{rrrrr} & 5, & 0 & 0 & 1 \\ - & & 8 & 9 & 7 \\ \hline \end{array}$
2	**Go to the lowest place-value column** that has not been subtracted. If the bottom number is larger than the top number, it is necessary to borrow from the next place value; otherwise, subtract the bottom number from the top number. So, we need to borrow from the "tens" place, but there are no units in the "tens" place. Moving left, there are also not any units in the "hundreds" place. **We continue moving left until we find a place with units from which to borrow.** We first borrow a unit from "thousands" place, and put it the "hundreds" place. Now we are able to borrow a unit from the "hundreds" place and put it in the "tens" place. Finally, we can borrow a unit from the "tens" place and put it in the "ones" place.	$\begin{array}{rrrrr} & & 9 & 9 & \\ & 4 & \cancel{10} & \cancel{10} & 11 \\ & \cancel{5,} & \cancel{0} & \cancel{0} & \cancel{1} \\ - & & 8 & 9 & 7 \\ \hline \end{array}$
3	Now we can subtract the 7 in the "ones" place from 11 in the "ones" place.	$\begin{array}{rrrrr} & & 9 & 9 & \\ & 4 & \cancel{10} & \cancel{10} & 11 \\ & \cancel{5,} & \cancel{0} & \cancel{0} & \cancel{1} \\ - & & 8 & 9 & 7 \\ \hline & & & & 4 \end{array}$
4	Return to Step 2 and complete the subtraction process.	$\begin{array}{rrrrr} & & 9 & 9 & \\ & 4 & \cancel{10} & \cancel{10} & 11 \\ & \cancel{5,} & \cancel{0} & \cancel{0} & \cancel{1} \\ - & & 8 & 9 & 7 \\ \hline & 4, & 1 & 0 & 4 \end{array}$

HOW TO SUBTRACT NUMBERS (continued)

Subtracting a sequence of numbers

The subtraction procedure is the same. Simply subtract the next number from the previous result.

Example: Subtract these numbers: 256 – 50 – 78 – 14.

First:	256	Next:	206	Next:	128
	– 50		– 78		– 14
	206		128		114 ← **Answer**

CHECKING SUBTRACTION ANSWERS

Checking your work: use addition

No one performs every calculation perfectly every time, even with a calculator. For this reason, it is important to know how to check your work. To check subtraction:

- Add the answer to the smaller number in the subtraction.
- If this total is equal to the larger number, the subtraction is correct.

Example

5,001	To check:	4,104
– 897		+ 897
4,104		5,001 ← **Subtraction is correct**

PRACTICE

REINFORCEMENT PROBLEM: SUBTRACTION

1. - Subtract the numbers shown in the left column of the table and show your answers in the right column.
   - Check by addition.

Subtract the following numbers …	Answer	Check
a. 129 – 59		
b. 2,073 – 1,891		
c. 747 – 599		
d. 244 – 185		
e. 35,402 – 29,515		
f. 147,459 – 127,456		
g. 8,757,930 – 4,899,348		
h. 19,824 – 9,009		
i. 54,741 – 44,230		
j. 12,300 – 7,999		
k. 4,002 – 599		
l. 5,005 – 4,788		

SOLUTIONS

REINFORCEMENT PROBLEM: SUBTRACTION

1.

Subtract the following numbers …	Answer	Check
a. 129 – 59	70	70 + 59 = 129
b. 2,073 – 1,891	182	182 + 1,891 = 2,073
c. 747 – 599	148	148 + 599 = 747
d. 244 – 185	59	59 + 185 = 244
e. 35,402 – 29,515	5,887	5,887 + 29,515 = 35,402
f. 147,459 – 127,456	20,003	20,003 + 127,456 = 147,459
g. 8,757,930 – 4,899,348	3,858,582	3,858,582 + 4,899,348 = 8,757,930
h. 19,824 – 9,009	10,815	10,815 + 9,009 = 19,824
i. 54,741 – 44,230	10,511	10,511 + 44,230 = 54,741
j. 12,300 – 7,999	4,301	4,301 + 7,999 = 12,300
k. 4,002 – 599	3,403	3,403 + 599 = 4,002
l. 5,005 – 4,788	217	217 + 4,788 = 5,005

HOW TO MULTIPLY NUMBERS

Introduction

Multiplication is really a fast way of doing addition, in the situation where there are numbers that are repeated. For example, suppose that you have seven boxes and each box contains five units of merchandise. To find the total units of merchandise, you could repeatedly add the number five until you have added it seven times:

$$\begin{array}{r} {}^{3}5 \\ 5 \\ 5 \\ 5 \\ 5 \\ 5 \\ \underline{5} \\ 35 \end{array}$$

It is much easier to multiply five times seven:

$$\begin{array}{r} 5 \\ \times \underline{\ 7} \\ 35 \end{array}$$

Terminology

The following terms are normally used when referring to multiplication:

- Each of the numbers being multiplied are called **factors**.
- The bottom factor is usually called the **multiplier**.
- The answer is called the **product**.

$$\begin{array}{lrl} \textbf{factors} \rightarrow & 5 & \\ \textbf{factors} \rightarrow & \times \underline{\ 7} & \textbf{multiplier} \\ & 35 & \textbf{product} \end{array}$$

Showing multiplication

Multiplication can be shown in several ways. The most common way to indicate multiplication is using the "×" or "times" sign:

$$\begin{array}{r} 7 \\ \times \underline{\ 5} \\ 35 \end{array} \qquad \text{or: } 7 \times 5 = 35$$

Other ways to show multiplication are: $7 \bullet 5 = 35$ or: $(7)\,(5) = 35$

HOW TO MULTIPLY NUMBERS (continued)

Memorizing multiplication for two single-digit numbers

If you have any trouble remembering how to multiply two single-digit numbers (such as “7 times 5” or “9 times 6”), you must get the basic multiples memorized. These multiples are found in the “Multiplication table” on page 52. It is not very difficult—you can make games out of the practice—and it is absolutely necessary. If you still need to memorize the multiples, do it.

How to multiply when one number is a single digit

The following table shows you how to multiply 135 by 7.

<table>
<tr><th>Step</th><th colspan="2">Action</th><th>Example</th></tr>
<tr><td rowspan="4">2</td><td colspan="2">Multiply the right digit of the multiplier with right digit of the top factor (7 times 5 is 35).</td><td>3
1 3 5
× 7
5</td></tr>
<tr><th>If …</th><th colspan="2">Then, under the line …</th></tr>
<tr><td>the product is 9 or less,</td><td colspan="2">write the product in the same place-value alignment as the digit in the multiplier.</td></tr>
<tr><td>the product is greater than 9,</td><td colspan="2">• write the right digit (5) of the product (35) in the same place-value column as the digit in the multiplier.
• write the left digit (3) of the product above the next number in the top factor.</td></tr>
<tr><td>3</td><td colspan="2">• Moving to the left, multiply the next digit in the top factor by the same digit in the multiplier (3 times 7 is 21).
• Then add any number written above the top digit coming from the previous calculation (21 plus 3 is 24).</td><td>2 3
1 3 5
× 7
4 5</td></tr>
</table>

(table continued on next page)

HOW TO MULTIPLY NUMBERS (continued)

Step	Action	Example
3 (cont.)	***If …***	***Then, under the line …***
	the product is 9 or less,	• write the result in the next available place-value column to the left of the previous number in the answer.
	the product is greater than 9,	• write the *right* digit (**4**) of the product (**24**) in the next available place-value column to the left of the previous number. • write the *left* digit (**2**) of the product above the next number in the top factor.
4	Moving to the left in the top factor, **continue** using the same procedures in Step 3 until all the digits in the top factor have been multiplied by the multiplier.	$\begin{array}{rrrr} & \leftarrow & & \\ & {\scriptstyle 2} & {\scriptstyle 3} & \\ & 1 & 3 & 5 \\ \times & & & 7 \\ \hline & \mathbf{9} & 4 & 5 \end{array}$

When both factors have multiple digits

The following six steps show you how to multiply when each factor has two or more digits.

Example: Multiply 135 and 287.

Step	Action	Example
1	**Align the factors** so that the individual digits are in the correct place-value columns. The factor with the fewest digits is the multiplier. Put it on the bottom, above a line. (In this case, either number can be on the bottom, because they both have the same number of digits.)	$\begin{array}{rrrr} & 1 & 3 & 5 \\ \times & 2 & 8 & 7 \\ \hline \end{array}$
2	**Complete the multiplication of the right digit** in the multiplier. (Use the method shown in the previous table.)	$\begin{array}{rrrr} & {\scriptstyle 2} & {\scriptstyle 3} & \\ & 1 & 3 & 5 \\ \times & 2 & 8 & 7 \\ \hline & 9 & 4 & 5 \end{array}$

(table continued on next page)

HOW TO MULTIPLY NUMBERS (continued)

<table>
<tr><th>Step</th><th colspan="2">Action</th><th>Example</th></tr>
<tr><td rowspan="4">3</td><td colspan="2">Move left to the next digit in the multiplier (8). Multiply this by the right digit in the top factor (5) (5 times 8 is 40).</td><td>2   4 ~~3~~
1   3   5
×   2   8   7
9   4   5
0
Notice the alignment</td></tr>
<tr><th>If …</th><th colspan="2">Then, under the line …</th></tr>
<tr><td>the product is 9 or less,</td><td colspan="2">write the product in the same place-value column as the digit you are using in the multiplier.</td></tr>
<tr><td>the product is greater than 9,</td><td colspan="2">• write the right digit (0) of the product (40) in the same place-value alignment as the digit you are using in the multiplier.
• write the left digit (4) of the product above the next number in the top factor. (Cross out any other number previously written here.)</td></tr>
<tr><td rowspan="5">4</td><td colspan="2">Using the same multiplier, multiply the next digit to the left in top factor (3 times 8 is 24). Then add the new number written above the top digit from the previous calculation (24 plus 4 is 28).</td><td>2 ~~2~~ 4 ~~3~~
1   3   5
×   2   8   7
9   4   5
8   0</td></tr>
<tr><th>If …</th><th colspan="2">Then, under the line …</th></tr>
<tr><td>the product is 9 or less,</td><td colspan="2">write the product in the next available place-value column to the left of the previous number.</td></tr>
<tr><td>the product is greater than 9,</td><td colspan="2">• write the right digit (8) of the product (28) in the next available place-value column to the left of the previous number.
• write the left digit (2) of the product above the next number in the top factor.</td></tr>
<tr><td colspan="2">Using the rules, finish multiplying all the digits in the top factor by the same digit (8) in the multiplier.</td><td>2 ~~2~~ 4 ~~3~~
1   3   5
×   2   8   7
9   4   5
1   0   8   0</td></tr>
</table>

(table continued on next page)

HOW TO MULTIPLY NUMBERS (continued)

Step	Action	Example
5	Return to Steps 3 and 4 until all the digits in the top factor have been multiplied by all the digits in the multiplier.	$\begin{array}{rrrrrr} & & & & {\scriptstyle 1} & \\ & & & {\scriptstyle 2\,\cancel{2}} & {\scriptstyle \cancel{4}\,\cancel{3}} & \\ & & & 1 & 3 & 5 \\ \times & & & 2 & 8 & 7 \\ \hline & & & 9 & 4 & 5 \\ & 1 & 0 & 8 & 0 & \\ & 2 & 7 & 0 & & \end{array}$
6	Add the place-value columns under the line to obtain the answer (in this example: 38,745). *Note:* The answer is called the **product**.	$\begin{array}{rrrrrr} & & & & {\scriptstyle 1} & \\ & & & {\scriptstyle 2\,\cancel{2}} & {\scriptstyle \cancel{4}\,\cancel{3}} & \\ & & & 1 & 3 & 5 \\ \times & & & 2 & 8 & 7 \\ \hline & & & 9 & 4 & 5 \\ & 1 & 0 & 8 & 0 & \\ & 2 & 7 & 0 & & \\ \hline & 3 & 8, & 7 & 4 & 5 \end{array}$

When the multiplier has zeroes

If the multiplier is a number that contains one or more zeroes, do not multiply by a zero. Instead, put a zero under the line in the same place-value column as the zero in the multiplier. Then move left in the multiplier to the next number that is not a zero, and follow normal procedures.

Example 1: Multiply 573 by 250. The product is: 143,250.	$\begin{array}{rrrrrrr} & & & & {\scriptstyle 1\,\cancel{3}} & {\scriptstyle 1} & \\ & & & & 5 & 7 & 3 \\ \times & & & & 2 & 5 & 0 \\ \hline & & & & & & \mathbf{0} \\ & & 2 & 8 & 6 & 5 & \\ & 1 & 1 & 4 & 6 & & \\ \hline & 1 & 4 & 3, & 2 & 5 & 0 \end{array}$

HOW TO MULTIPLY NUMBERS (continued)

Example 2: Multiply 573 by 205.

$$
\begin{array}{rcccccc}
 & & & & \overset{1\,\cancel{3}}{5} & \overset{1}{7} & 3 \\
\times & & & & 2 & 0 & 5 \\
\hline
 & & & 2 & 8 & 6 & 5 \\
 & & & & & \mathbf{0} & \\
 & 1 & 1 & 4 & 6 & & \\
\hline
 & 1 & 1 & 7, & 4 & 6 & 5
\end{array}
$$

The product is: 117,465.

Shortcut: when there are zeroes at the end of factors

Whenever a factor ends in zero, you can drop the zero before you multiply, then do the multiplication with the remaining digits. Attach the same number of zeroes to your answer as the number of zeroes you dropped. This saves time and reduces error.

Example: Multiply 4,500 and 320.

You can drop a total of three zeroes before you multiply. You will then attach three zeroes back to the product.

Answer: 1,440,000.

$$
\begin{array}{rcccccc}
 & & 4 & 5 & & & \\
\times & & 3 & 2 & & & \\
\hline
 & & 9 & 0 & & & \\
1 & 3 & 5 & & & & \\
\hline
1, & 4 & 4 & 0, & \mathbf{0} & \mathbf{0} & \mathbf{0}
\end{array}
$$

Attach zeroes

HOW TO MULTIPLY NUMBERS (continued)

Characteristics of multiplication

Numbers can be multiplied in any order and the answer is the same:
- Multiplying 7 × 25 = 175
- Multiplying 25 × 7 = 175

Numbers can be multiplied in any groups and the answer is the same:
- Multiplying (9 × 48) × 22 is 432 × 22 = 9,504
- Multiplying 9 × (48 × 22) is 9 × 1,056 = 9,504

The product of any number and zero is always zero:
- Multiplying 245 × 0 = 0

The product of any number and 1 is always the number itself:
- Multiplying 245 × 1 = 245

Multiplying a sequence of numbers

The multiplication procedure is the same. Simply multiply the next number times the previous result, until there are no more numbers to multiply.

Example: Multiply the following numbers: 27 × 50 × 219

- First, multiply 27 times 50: 27 × 50 = 1,350
- Then multiply by the next number: 1,350 × 219 = 295,650

PRACTICE

SOLUTIONS FOR MULTIPLICATION BEGIN ON PAGE 30.

REINFORCEMENT PROBLEMS: MULTIPLICATION

1. In the table below, complete each indicated operation and write your answer in the “Answer” column next to the operation.

Multiply the following numbers …	Answer
a. 255 × 8	
b. 255 × 38	
c. 255 × 938	
d. 255 × 2,938	
e. 207 × 412	
f. 2,400 × 3,500	
g. 909 × 303	
h. 684 × 729	
i. 250 × 190	
j. 1,877 × 300	
k. 72 × 211	
l. 952 × 743	
m. 822 × 3,005	
n. 215 × 49	

2. a. If professor Gillis grades 4 tests per hour, how many tests can he grade in 8 hours?

   b. A business collects $1 of sales tax for every $15 of sales. If the sales tax collection was $9,000, what was the total amount of sales?

   c. The labor cost for manufacturing a computer is $52 per computer. What is the total labor manufacturing cost if 850 computers are manufactured this week?

SOLUTIONS

PRACTICE QUESTIONS FOR MULTIPLICATION BEGIN ON PAGE 29.

REINFORCEMENT PROBLEMS: MULTIPLICATION

1.

Operation	Answer
a. 255 × 8	2,040
b. 255 × 38	9,690
c. 255 × 938	239,190
d. 255 × 2,938	749,190
e. 207 × 412	85,284
f. 2,400 × 3,500	8,400,000
g. 909 × 303	275,427
h. 684 × 729	498,636
i. 250 × 190	47,500
j. 1,877 × 300	563,100
k. 72 × 211	15,192
l. 952 × 743	707,336
m. 822 × 3,005	2,470,110
n. 215 × 49	10,535

2. a. 4 tests × 8 = 32 total tests
   b. $15 × $9,000 = $135,000 total sales
   c. $52 labor × 850 = $44,200 total labor cost

HOW TO DIVIDE NUMBERS

Introduction

Division is the process of finding out how many times one number is contained in another number. It is the opposite of multiplication.

Example

Suppose that you have a loaded container that can hold 3,750 pounds, and the container is full. Each unit of your merchandise weighs 15 pounds. How many units of merchandise are in the container? In other words, we are asking how many times is the number 15 (for each unit) contained in the number 3,750.

Terminology

The following terms are normally used when referring to division:

- The "container" number is called the **dividend** (3,750 in this example).
- The number being contained is called the **divisor** (15 in this example).
- The result—the answer—is called the **quotient** (250 units in this example).

Showing division

Division can be shown in several ways. The most common way to indicate division are the following signs: ÷, or /, or ——, or $\overline{)}$

$$\underset{\text{dividend}}{3{,}750} \div \underset{\text{divisor}}{15} = \underset{\text{quotient}}{250}$$

or, 3,750 / 15 = 250

or, $\dfrac{3{,}750}{15} = 250$

or, $15\,\overline{)\,3{,}750} = 250$

How to say division calculations

There are two equally correct ways of verbalizing division calculations:

- You can say the dividend amount, then say "divided by" the divisor amount ("3,750 divided by 15 equals 250") or,
- You can say the divisor amount, then say "divided into" the dividend amount ("15 divided into 3,750 equals 250").

HOW TO DIVIDE NUMBERS (continued)

How to do division The table below shows the seven steps to use when you want to divide.

Example: Divide 4,259 by 23.

Step	Action	Example
1	Write the **dividend** inside a bracket, and write the **divisor** in front of the dividend. Ignore any commas.	$23\overline{)\,4259}$
2	Moving from left to right in the dividend, identify a partial dividend that is equal to or greater than the divisor, using the least possible digits. *Example:* "4" would not be correct because it is not equal to or greater than the divisor (23). "425" would not be correct, because a number with fewer digits is available (42), that is still equal to or greater than the divisor.	$23\overline{)\,\mathbf{42}59}$
3	Determine the **largest multiplier** of the divisor that will not result in a product greater than the partial dividend (here, not exceeding 42). Write this multiplier above the line, in the same place-value column as the right digit of the partial dividend. *Note:* Finding the correct multiplier may require a little trial and error. For example, 2 × 23 = 46, so 2 is too big of a multiplier because 46 exceeds 42.	**multiplier** $\begin{array}{r} \mathbf{1} \\ 23\overline{)\,4259} \end{array}$
4	Write the **product** of the multiplier times the divisor under the partial dividend (1 × 23 = 23). Be sure that the place values are aligned correctly.	$\begin{array}{r} 1 \\ 23\overline{)\,4259} \\ \mathbf{23} \end{array}$
5	Subtract the product you just wrote down from the partial dividend. *Check:* The **difference** must be *less than the divisor.*	$\begin{array}{r} 1 \\ 23\overline{)\,4259} \\ \underline{23} \\ \mathbf{19} \end{array}$ **difference**

(table continued on next page)

HOW TO DIVIDE NUMBERS (continued)

Step	Action	Example
6	Create a new partial dividend. Bring down from the dividend, to the right of the previous partial dividend, the fewest numbers needed to make a new partial dividend that is equal to or greater than the divisor.	1 2 3 | 4 2 5 9 2 3 ↓ **1 9 5**
7	Return to Step 3 and repeat the procedures until there are no more digits left to bring down from the dividend. If there is a **remainder**, write it in brackets next to the quotient and label it as "remainder." (This means that the quotient contained slightly more than 185 units of 23.) The quotient (answer) is 185, with a remainder of 4.	1 8 5 **(4) remainder** 2 3 | 4 2 5 9 2 3 ↓ 1 9 5 1 8 4 1 1 9 1 1 5 **4**

Rule: when there is zero in a quotient

You will need to place a zero in the quotient whenever a digit brought down from the dividend does not make the partial dividend equal to or larger than the divisor.

Example: Calculate 67,320 / 33.

Step	Action	Example
1	Write the dividend inside a bracket, and write the divisor in front of the dividend. Ignore any commas.	3 3 | 6 7 3 2 0
2	Identify the partial dividend.	3 3 | 6 7 3 2 0
3	Determine the largest multiplier of the divisor that will not result in a product greater than the partial dividend.	2 3 3 | 6 7 3 2 0
4	Write the product of the multiplier times the divisor under the partial dividend.	2 3 3 | 6 7 3 2 0 6 6

(table continued on next page)

HOW TO DIVIDE NUMBERS (continued)

Step	Action	Example
5	Subtract the product you just wrote down from the partial dividend.	2 33 ⟌ 67320 66 1
6	Create a new partial dividend. In this example, *two digits* must be brought down from the dividend, because bringing down only the 3 makes 13, which is not equal to or greater than the divisor of 33. *Rule:* You must place a **zero** above any number brought down that does not make the partial dividend equal to or greater than the divisor.	**zero needed** 2 **0** 33 ⟌ 67320 66 ↓ ↓ 132
7	Return to Step 3 and repeat the procedures until there are no more digits left to bring down from the dividend. The quotient (answer) is 2,040.	2040 33 ⟌ 67320 66 ↓ ↓ 132 132 **no remainder** 0

Shortcut: when divisor and dividend end in zero

Whenever both the dividend and divisor end in zero, drop an equal number of zeroes from the ends of both numbers. Then do the division with the remaining digits. This saves time and reduces error.

Example: Evaluate 5,500 ÷ 50	110 5 ⟌ 550 5 5 5
Example: Evaluate 11,790 / 170	69 **(6) remainder** 17 ⟌ 1179 102 159 153 **6**

HOW TO DIVIDE NUMBERS (continued)

Example: Evaluate $\frac{4{,}200}{1{,}000}$	$10\overline{)42}$ = 4 **(2) remainder** $42 - 40 = \mathbf{2}$

CHECKING DIVISION ANSWERS

Checking your work: use multiplication

No one performs every calculation perfectly every time, even with a calculator. For this reason, it is important to know how to check your work. To check division:

- **Step 1:** Multiply the quotient times the divisor.
- **Step 2:** If there is a remainder, add the remainder to the product in Step 1.

The answer will equal the dividend if the division is correct.

Examples

550 ÷ 5 = 110	Check: 110 × 5 = 550	**Quotient is correct**
1,179 / 17 = 69 r6	Check: 69 × 17 = 1,173 + 6 1,179	**Quotient is correct**
322 ÷ 11 = 29 r3	Check: 29 × 11 = 319 + 3 322	**Quotient is correct**

Note: The letter “r” indicates a remainder.

“BY” AND “INTO” ARE TWO IMPORTANT WORDS

Introduction

Here is a very common mistake when reading and listening to an explanation about division operations: confusing the meaning of “divide by” and “divide into.”

"BY" AND "INTO" ARE TWO IMPORTANT WORDS (continued)

"Divide by"

When you read or hear the expression "divide *by*" followed by a number, that number refers to *the divisor.*

Examples:
- "Divide 350 by 25" means 350 ÷ 25.
- "1,300 is divided by 200" means 1,300 ÷ 200.

"Divide into"

When you read or hear the expression "divide *into*" followed by a number, that number refers to *the dividend*.

Examples:
- "Divide 40 into 850" means 850 ÷ 40.
- "1,400 is divided into 12,000" means 12,000 ÷ 1,400.

USING DIVISION IN "RATE" PROBLEMS

Definition: rate

The word **rate** refers to the comparison of two amounts of different things.

> *Note:* Sometimes the comparison is expressed as a percent. This is discussed in greater detail later on, page 73.

Rate per unit

Very often, rate information is given in a way that compares an amount of something to *one unit* of a different kind of thing. This is the clearest way to show a rate, and is often called "rate per unit."

Examples

The table below shows some examples of how rate can be expressed per unit.

Example	This amount …	is being compared to this unit …
"Marty earns $15 per hour."	15 (dollars)	1 (hour)
"In one hour the train can travel 80 miles."	80 (miles)	1 (hour)
"The car will travel 22 miles per gallon."	22 (miles)	1 (gallon of gas)
"For every dollar of profit the business needed $17 of sales."	17 (dollars of sales)	1 (dollar of profit)

USING DIVISION IN "RATE" PROBLEMS (continued)

Express totals as a rate per unit

Frequently, you may have information that shows two total amounts, but which would be very helpful to you if they would be compared as a rate per unit.

Examples

- Marty earned $255 for 17 hours of work. Marty would like to know how much he is being paid for each hour that he works (in other words, the rate of pay per hour).
- This year, you earned $75,200 and paid $18,800 in taxes. It would be very revealing to find out how much income you had to earn for every dollar in tax that you paid (in other words, the rate of income earned per tax dollar paid).
- You traveled 418 miles and used 19 gallons of gasoline. To check the efficiency of your car, you would like to know the miles traveled for each gallon of gasoline used (in other words, the rate of miles per gallon).

Procedure: calculate rate per unit

To compare two total amounts as a rate per unit:
- Identify the amount you want to be the unit of reference.
- Divide this amount into the other number.

Examples

Converting Marty's pay to a rate per hour:
- To obtain the pay *per hour*, hours become the unit of reference.
- Divide: $255 ÷ 17 hours = $15 wages per hour

Converting your income to an amount per dollar of tax paid:
- To obtain the income *per tax dollar*, the tax is the unit of reference.
- Divide: $75,200 ÷ $18,800 = $4 of income per dollar of tax.

Converting your travel mileage to a rate per gallon:
- To obtain the rate *per gallon*, gallons become the unit of reference.
- Divide: 418 miles ÷ 19 gallons = 22 miles per gallon.

Using the "/ "

Sometimes rate is expressed by using the "/ " sign.

Example:
"Marty earns $15/hour.

MORE USES FOR RATE PER UNIT

Two more uses

Once you know the rate per unit information, it can also be used in two more very handy ways.

- Specify any number of reference units to find a total.
- Specify a total to find the number of reference units.

Procedure: calculate a total from a specified number of units

Procedure:
- Specify an amount for the number of units.
- Multiply this amount by the rate per unit.

Example #1: "Marty earns $15 per hour. How much will he earn after 12 hours of work?"
- Specified amount of units: 12 (hours)
- 12 hours × $15 = $180 (total $)

Example #2: "Your gas mileage is 22 miles per gallon. How far could you travel on 50 gallons?"
- Specified amount of units: 50 (gallons)
- 50 gallons × 22 = 1,100 miles (total miles)

Procedure: calculate number of units from a specified total

Sometimes you need to find out the number of units of the reference item that will result from a specified total. After you know the rate per unit, do this:

Procedure:
- Specify a new "what if?" total.
- Divide this by the rate per unit.

Example #1: "Marty earns $15 per hour (rate per unit). How many hours will he have to work to earn $450?"
- Specified new total amount: $450
- $450 ÷ $15 per hour = 30 hours (amount of reference units—hours)

Example #2: "Your gas mileage is 22 miles per gallon (rate per unit). If you traveled 8,800 miles, how many gallons did you use?"
- Specified new total amount: 8,800 miles
- 8,800 miles ÷ 22 miles/gallon = 400 gallons (amount of reference units—gallons)

PRACTICE

SOLUTIONS FOR DIVISION BEGIN ON PAGE 40.

REINFORCEMENT PROBLEMS: DIVISION

1. In the table below, complete each indicated operation and write your answer in the "Answer" column next to the operation. Check your answers.

Divide the following numbers …	Answer	Check
a. 350 ÷ 25		
b. 2,576 ÷ 23		
c. 1,560 ÷ 15		
d. 1,218 ÷ 52		
e. 8,450 ÷ 241		
f. 11,250 ÷ 45		
g. 15,000 ÷ 3,000		
h. 3,200 ÷ 150		
i. 35,734 ÷ 17		
j. 79,184 ÷ 112		
k. 592 ÷ 38		
l. 4,200 ÷ 30		
m. 12,015 ÷ 252		
n 980 ÷ 50		

2. a. Myer Company paid $8,250 for 750 pounds of raw materials. What is the cost per pound?

   b. You paid $135 for 9 gallons of paint. What is the cost per gallon?

   c. A storage tank is leaking toxic chemicals at the rate of 2 gallons per week. How many gallons will it leak after 52 weeks?

   d. Our company sells Internet software packages for $35 per package. How many packages must be sold in order to have $42,000 of revenue?

   e. Our company makes a profit of $50 for every computer that we sell. We had a profit of $375,000 this month. How many computers were sold?

   f. If Lo Landscaping Service can mow the lawns for 3 homes in 1 hour, how many hours will be required to mow the lawns for 90 homes?

SOLUTIONS

PRACTICE QUESTIONS FOR DIVISION BEGIN ON PAGE 39.

REINFORCEMENT PROBLEMS: DIVISION

1.

Operation	Answer	Check
a. 350 ÷ 25	14	14 × 25 = 350
b. 2,576 ÷ 23	112	112 × 23 = 2,576
c. 1,560 ÷ 15	104	104 × 15 = 1,560
d. 1,218 ÷ 52	23 r22	23 × 52 = 1,196 + 22 = 1,218
e. 8,450 ÷ 241	35 r15	35 × 241 = 8,435 + 15 = 8,450
f. 11,250 ÷ 45	250	250 × 45 = 11,250
g. 15,000 ÷ 3,000	5	5 × 3 = 15
h. 3,200 ÷ 150	21 r5	21 × 15 = 315 + 5 = 320
i. 35,734 ÷ 17	2,102	2,102 × 17 = 35,734
j. 79,184 ÷ 112	707	707 × 112 = 79,184
k. 592 ÷ 38	15 r22	15 × 38 = 570 + 22 = 592
l. 4,200 ÷ 30	140	140 × 3 = 420
m. 12,015 ÷ 252	47 r171	47 × 252 = 11,844 + 171 = 12,015
n. 980 ÷ 50	19 r3	19 × 5 = 95 + 3 = 98

Note: The letter "r" indicates a remainder. Don't forget to use the shortcut of dropping zeroes (items g, h, l, and n)!

2. a. $8,250 / 750 pounds = $11 per pound
   b. $135 / 9 gallons = $15 per gallon
   c. 52 weeks × 2 gallons/week = 104 gallons
   d. $42,000 / $35 = 1,200 packages
   e. $375,000 / $50 = 7,500 computers
   f. 90 homes / 3 homes/hour = 30 hours

ROUNDING NUMBERS

What is "rounding"?

Rounding is the technique of simplifying a number by replacing a designated number of digits with zero.

The purpose of rounding

When a number is rounded, it is easier to use and remember, even though some precision is lost.

It is easier to remember …	than …
Rounded	**Not rounded or rounded less**
3,290	3,287
5,300	5,290
8,000	8,300

When is rounding used?

Rounding is very useful. The most important uses of rounding are:

- checking the reasonableness of calculations to avoid large errors
- making quick estimates of values
- simplifying financial and economic reports used for decision-making

Rounding procedure

The following three steps are used to round a number:

<table>
<tr><th>Step</th><th colspan="2">Action</th></tr>
<tr><td>1</td><td colspan="2">In the number selected, determine which place-value digit is to be rounded.

Note: This information is often specified in a problem, or by an instructor or supervisor.</td></tr>
<tr><td rowspan="3">2</td><td>If …</td><td>Then …</td></tr>
<tr><td>the digit to right of the digit to be rounded is 5 or greater,</td><td>add 1 to the digit to be rounded.</td></tr>
<tr><td>the digit to the right of the digit to be rounded is less that 5,</td><td>the digit to be rounded remains unchanged.</td></tr>
<tr><td>3</td><td colspan="2">Replace each digit to the right of the digit to be rounded with a zero.</td></tr>
</table>

ROUNDING NUMBERS (continued)

Examples The following table illustrates examples of rounded numbers.

Rounding Specification	Rounded Number
Round 4,758 to the "tens" place	4,760
Round 4,758 to the "hundreds" place	4,800
Round 4,758 to the greatest place-value digit	5,000
Round 212, 954 to the "hundreds" place	213,000
Round 212, 954 to the "thousands" place	213,000
Round 212, 954 to the "ten thousands" place	210,000
Round 212, 954 to the "hundred thousands" place	200,000

PRACTICE

REINFORCEMENT PROBLEM: ROUNDING NUMBERS

1. In each column, write the rounded number to the place value indicated. Use the first number as an example.

	Round to this place-value …				
Number	**tens**	**hundreds**	**thousands**	**ten thousands**	**greatest place-value**
1,636	1,640	1,600	2,000	–0–	2,000
6,250					
15,975					
9,025,479					
821					
14,307					
5,825					
11,197					
272,889					

SOLUTIONS

REINFORCEMENT PROBLEM: ROUNDING NUMBERS

1.

	Round to this place-value …				
Number	**tens**	**hundreds**	**thousands**	**ten thousands**	**greatest place-value**
1,636	1,640	1,600	2,000	–0–	2,000
6,250	6,250	6,300	6,000	10,000	6,000
15,975	15,980	16,000	16,000	20,000	20,000
9,025,479	9,025,480	9,025,500	9,025,000	9,030,000	9,000,000
821	820	800	–0–	–0–	800
14,307	14,310	14,300	14,000	10,000	10,000
5,825	5,830	5,800	6,000	10,000	6,000
11,197	11,200	11,200	11,000	10,000	10,000
272,889	272,890	272,900	273,000	270,000	300,000

CHECKING THE REASONABLENESS OF AN ANSWER

Introduction

The ability to quickly check the reasonableness of an answer is a very important skill. It is especially useful in financial, accounting, and business calculations where money is involved. This is true **even if you use a calculator**. Large and embarrassing errors can be made on a calculator more easily than in hand-prepared calculations.

"Checking reasonableness" means …

To "check reasonableness" means to do a quick estimate calculation for the purpose of avoiding large errors. You are checking that the answer is approximately the size that it should be. *This may be done before or after you do the exact calculation.*

How to check for reasonableness

The following procedure applies to addition, subtraction, multiplication, or division:

<table>
<tr><th>Step</th><th colspan="2">Action</th></tr>
<tr><td>1</td><td colspan="2">Round all numbers to the greatest place-value digit.</td></tr>
<tr><td>2</td><td colspan="2">Using the rounded numbers, perform whatever calculation is required. This is your estimate.</td></tr>
<tr><td rowspan="4">3</td><td colspan="2">Compare the estimate from Step 2 to the exact answer.</td></tr>
<tr><td>If …</td><td>Then …</td></tr>
<tr><td>the estimate is "reasonably close" to the exact answer,</td><td>there are probably no large errors.</td></tr>
<tr><td>the estimate is very different than the exact answer,</td><td>recalculate for a new exact answer.</td></tr>
</table>

CHECKING THE REASONABLENESS OF AN ANSWER (continued)

Examples The following examples show you how to check for reasonableness:

Calculation	**Numbers**	**Exact Calculation**	**Reasonableness Check**
addition	add: 4,579 + 1,926 + 11,842 + 3,875 rounded: 5,000 + 2,000 + 10,000 + 4,000	4,579 1,926 11,842 + 3,875 ——— 22,222	5,000 2,000 10,000 + 4,000 ——— 21,000
subtraction	subtract: 313,807 – 109,482 rounded: 300,000 – 100,000	313,807 – 109,482 ——— 204,325	300,000 – 100,000 ——— 200,000
multiplication	multiply: 17,947 × 582 rounded: 20,000 × 600	17,947 × 582 ——— 35894 143576 89735 ——— 10,445,154	**Using multiplication shortcut:** 2 × 6 ——— 12,000,000 **(Zeroes are dropped and then added back to the answer)**
division	divide: 39,850 / 420 rounded: 40,000 / 400	94 420) 39850 3780 2050 1680 **Remainder** 370	**Using division shortcut:** 100 4) 400 4 00 **(An equal number of zeroes are dropped from each number)**

PRACTICE

REINFORCEMENT PROBLEM: DOING A REASONABLENESS CHECK

1. Complete the following table:

Calculation	Reasonableness Check	Exact Answer
4,812 × 3,499		
92,100 ÷ 217		
453,191 ÷ 72,647		
807 × 354		
7,985 × 8,722		

SOLUTIONS

REINFORCEMENT PROBLEM: DOING A REASONABLENESS CHECK

1.

Calculation	Reasonableness Check	Exact Answer
4,812 × 3,499	15,000,000	16,837,188
92,100 ÷ 217	450	424 r92
453,191 ÷ 72,647	7 with remainder	6 r17,309
807 × 354	320,000	285,678
7,985 × 8,722	72,000,000	69,645,170

Note: The letter "r" indicates a remainder.

WHICH OPERATION DO I USE?

Introduction

After you have practiced doing the calculations for the four operations of addition, subtraction, multiplication, and division, you are ready to apply what you have learned. Sometimes in a problem, the calculation signs will be given to you, such as: 475 – 224, or 845 × 22. In these kinds of situations, the operation symbols make it clear what to do.

However, very frequently you will be given the facts of a situation and you will have to decide which of the four operations you will need to apply to the facts. In other words, you will have to analyze the facts, and then decide which operation is needed. The guidelines below summarize when to apply the four operations.

Addition

The following table shows how to identify situations that require addition. In all cases, the underlying idea is to find the total of different numbers.

Add to …	Examples
a. Find a total of different amounts	• Janice purchased items costing the following amounts: $4, $50, $122 and $435. What did she pay for all the purchases? ($104 + $50 + $122 + $435 = $711) • On the first three quizzes, Andrew scored 88, 79, and 91. What are his total points? (88 + 79 + 91 = 258)
b. Increase an amount by another amount	• Anselmo Company began the week with $10,532 in its savings account. During the week, it deposited $5,000 more into the account. How much does the company have at the end of the week? ($10,532 + $5,000 increase = $15,532) • Thuan Enterprises hired 112 new employees last year. How many employees does the company have now if it had 455 employees at the beginning of the year? (455 + 112 = 567)

WHICH OPERATION DO I USE? (continued)

Subtraction

The following table shows you how to identify situations that require subtraction. In all cases, the underlying idea is to calculate a numerical difference between two numbers.

Subtract to …	Examples
a. Find a difference between two amounts	• If the cost of a vacation to Hawaii is \$2,795 and the cost of a vacation to Bermuda is \$3,977, how much more does the vacation to Bermuda cost? (\$3,977 – \$2,795 = \$1,182)
b. Calculate a change	• Anselmo Company began the week with \$10,532 in the savings account. At the end of the week, the company had \$15,532 in the account. How much did the account change? (\$15,532 – \$10,532 = \$5,000) • If the total debts of Lowjewski Company at the beginning of the month were \$41,005, and \$12,300 at the end of the month, how much did the debts change? (\$41,005 – \$12,300 = \$28,705 decrease)
c. Find an excess or left-over amount	• Ames Company began the month with \$4,190 in the office supplies account. If \$3,850 of supplies were used during the month, how much is still in the account? (\$4,190 – \$3,850 = \$340) • If the total assets of Diamond Enterprises are \$443,250 and the total debts are \$215,480, what is the owner's equity? (\$443,250 – \$215,480 = \$227,770)
d. Find one part of a larger amount	• Senatobia Ventures spent \$212,000, including sales tax, to purchase computers. If the sales tax was \$19,000, what was the cost of the computers? (\$212,000 – \$19,000 = \$193,000) • Holmes Company spent \$500,000 to purchase land with a building and equipment. If the building was appraised at \$273,000 and the equipment was appraised at \$87,500, what was the cost of the land? (\$500,000 – \$273,000 – \$87,500 = \$139,500)

WHICH OPERATION DO I USE? (continued)

Multiplication

The following table shows you how to identify situations that require multiplication. In all cases, the underlying idea is that a single number is repeated a multiple number of times.

Multiply to …	Examples
a. Find the total of a repeated number or of an amount of equal parts	• On each day of the week, Monday through Friday, your business purchased 85 units of merchandise. What are the purchases for the week? (The amount of 85 units is repeated 5 times, so 85 × 5 = 425 units) • Denise has 10 equal containers and she pours 35 gallons of lemonade into each container. What is the total amount of lemonade?
b. Find the total for a specified number of units when you know the rate per unit	• Rasmussen Company pays wages at the rate of $450 per day. What total wages would be paid for 14 days? ("Per day" indicates that days are the units, and the rate per unit is $450. $450 × 14 = $6,300) • Hennepin Partnership purchased 500 shares of stock and each share cost $38. What is the total cost of the stock? ("Each share" indicates that shares are the units, and the rate per share is $38. $38 × 500 = $19,000)
c. Find the amount that is a "number of times" another number	• Dave earns **3 times as much** as John. If John earns $575 per week, how much does Dave earn per week? ($575 × 3 = $1,725) • If my car is traveling at 15 miles per hour and your car is traveling **4 times as fast** as my car, how fast is your car traveling? (15 mph × 4 = 60 mph) • The revenue of Jones Company is $90,000 and the revenue of Smith Company is **80%** of Jones Company. What is the revenue of Smith Company? ($90,000 × .8 = $72,000. In other words, Smith Company revenue is .8 times as much—less than—Jones Company.)

WHICH OPERATION DO I USE? (continued)

Division

The following table shows you how to identify situations that require division. In all cases, the underlying idea is that division finds a multiple that one number (the dividend) is of another number (the divisor).

Divide to …	Examples
a. Find the number for which some amount is a multiple (a "number of times") that number. *Note:* This is opposite of item "C" in multiplication (see page 49).	• Two aircraft are flying away from Dallas. The first aircraft is 1,200 miles away and this is 4 times as far away from Dallas as the second aircraft. How far away from Dallas is the second aircraft? (You want to find the number for which the first aircraft's distance of 1,200 miles is 4 times that number. 1,200 miles / 4 = 300 miles) • Dave earns $1,725 per week. If Dave earns 3 times as much as John, how much does John earn? (You want to find the number for which Dave's $1,725 is 3 times that number. $1,725 / 3 = $575)
b. Calculate a rate to compare an amount of something to a single unit of something else. *Note:* Rate is frequently described by using the word "per."	• Castlewood Company used up $185,000 over a period of 20 days. What was its rate of loss of cash per day? (You are comparing dollars to days. $185,000 / 20 **days** = 9,250 **dollars** per **day**) • D'Agostine's Catering Company used 105 pizzas to serve 420 people. What is the rate of people per pizza? (You are comparing people to pizzas. The rate per unit is 420 **people** / 105 **pizzas** = 4 **people** per **pizza**)
c. Find the size of equal parts when you know the total and the number of parts. *Note:* The answer can also be interpreted as a rate.	• If a rope is 24 feet long and divided into 8 pieces, how long is each piece? (24 **feet** / 8 **pieces** = 3 **feet** for each **piece**) • Denise used 35 gallons of lemonade to pour into 10 equal containers. How many gallons of lemonade does each container hold? (35 **gallons** / 10 **containers** = approximately 3.5 **gallons** in a **container**)
d. Find a number of units, when you know the total units and the rate per unit. *Note:* Because you are dividing by similar kinds of units, the answer is never a rate.	• Roswell Company manufactured and shipped boxes containing 12,000 pens. If each box contains 8 pens, how many boxes were shipped? (The rate per unit is 8 pens per box. 12,000 **pens** / 8 **pens** per box = 1,500 boxes) • Clovis Corporation budgeted a total of $300,000 to purchase new computers. If each computer costs $2,000, how many new computers can be purchased? (The rate per unit is $2,000 per computer. $300,000 / $2,000 per computer = 150 computers) • D'Agostine's Catering Company is expecting 420 people for a party. The company is providing pizza for the guests. If one pizza serves 4 people, how many pizzas will be needed? (The rate per unit is 4 people per pizza. 420 **people** / 4 **people** per pizza = 105 pizzas)

CALCULATING AN AVERAGE

What is an average?

An **average** is just a special case of finding an amount per unit. The only special thing about an average is that the number that you are dividing into (the dividend) is a total that results from various different events. The answer (the average) is interpreted as the single number that is most typical or representative of all the events.

Procedure

- Find the total that results from adding the amount of each different event.
- Divide the total by the number of events.

Example

Suppose that Sam had these four scores on accounting tests: 81, 92, 74, 88. What is his average score?

Answer: The events are the four tests.
- Add the scores: (81 + 92 + 75 + 88) = 336
- Divide by 4: 336 / 4 = 84

Therefore, 84 is Sam's average score. This is interpreted as being the amount that is most typical or representative of all the scores, when all the scores are equally important.

MULTIPLICATION TABLE

The table below shows the multiples for all numbers from 0 to 10. The numbers in the table are the products to memorize.

Example: To multiply 7 by 4, find 7 in left margin and move to the right on that line until you come to column that has 4 at the top. The number 28 is the product that is the answer.

X	0	1	2	3	4	5	6	7	8	9	10	
0	0	0	0	0	0	0	0	0	0	0	0	
1	0	1	2	3	4	5	6	7	8	9	10	(numbers increase in units of 1)
2	0	2	4	6	8	10	12	14	16	18	20	(numbers increase in units of 2)
3	0	3	6	9	12	15	18	21	24	27	30	(numbers increase in units of 3)
4	0	4	8	12	16	20	24	28	32	36	40	(numbers increase in units of 4)
5	0	5	10	15	20	25	30	35	40	45	50	(numbers increase in units of 5)
6	0	6	12	18	24	30	36	42	48	54	60	(numbers increase in units of 6)
7	0	7	14	21	28	35	42	49	56	63	70	(numbers increase in units of 7)
8	0	8	16	24	32	40	48	56	64	72	80	(numbers increase in units of 8)
9	0	9	18	27	36	45	54	63	72	81	90	(numbers increase in units of 9)
10	0	10	20	30	40	50	60	70	80	90	100	(numbers increase in units of 10)

PRACTICE

SOLUTIONS FOR CHOOSING THE CORRECT OPERATION BEGIN ON PAGE 55.

REINFORCEMENT PROBLEM: CHOOSING THE CORRECT OPERATION

1. a. For each of the separate situations in the table, place an "✘" in the correct box to indicate which type of calculation is required. (For some items, more than one calculation is needed.)

   b. Calculate the answer to each item after you complete part "a."

Item	Add.	Sub.	Mult.	Div.	Answer
1. Texarkana Company began the week with $15,404 in its checking account. There is $3,400 in the account at the end of the week. By how much did the account change?					
2. McLennan Business Supplies sold 125 computers, 1,127 notebooks, 16 printers, 5 fax machines, 10 modems, and 422 pens. How many items did the company sell?					
3. K.C.'s barbecue restaurant cooked 350 meals and used up 25 gallons of barbecue sauce. How many meals per gallon did the company cook?					
4. Austin Company is creating a computer software product that it wants to finish in one year. The company has estimated that approximately 34,000 labor hours will be required to complete the project. If one employee works an average of 2,000 hours per year, how many employees will be required for the project?					
5. Cisco Partnership has 12 employees that earn $23.00 per hour. If each employee works 8 hours per day, what is the daily total pay for all employees? What is the weekly pay for a 5-day week?					
6. Cerritos Enterprises made sales to customers totaling $478,300. If total expenses were $312,500, what was the profit?					
7. Fullerton Company had advertising expense of $147,000 and Merced Company had advertising expense of $12,250. The expense for Fullerton was how many times that of Merced?					
8. The net income of Savannah Company was 4 times the net income of Barnesville Company. If Savannah Company earned $81,000, how much did Barnesville earn?					
9. Martinez Enterprises produces 3 times as many items as Cerritos Enterprises. If Cerritos produced 4,200 units last year, how many did Martinez produce?					

P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E

PRACTICE

SOLUTIONS FOR CHOOSING THE CORRECT OPERATION BEGIN ON PAGE 55.

1, *continued*

Item	Add.	Sub.	Mult.	Div.	Answer
10. Chi paid $24,500 for a new car. This included $1,300 sales tax and $250 shipping. What was the cost of the car itself?					
11. Shawnee Company has 12 employees. The weekly payroll for all the employees is a total of $10,800. What is the average weekly wage of the employees?					
12. Sinclair Company had the following monthly sales in units: October: 3,205 units; November: 2,250 units; December: 2,249 units. Each unit sells for $14. What is the dollar sales for the three months?					
13. On May 1, El Camino Company had 417 units of supplies. During May, the company ordered and received 2,050 more units and used up 1,819 units. How many units were left on May 31?					
14. Manahoy Partnership constructed two buildings. One building cost $105,000 and the other building cost twice as much. What was the cost of both buildings?					
15. Manahoy Partnership has two partners that share profits. Partner A received $57,000 which was 3 times as much as Partner B. How much more money did Partner A earn?					
16. Dawson Company purchased $4,500 of office supplies. At the end of the month, an inventory shows that $715 of supplies are still on hand. How much was used up?					
17. Passaic Company has a net outflow of cash at the rate of $2,500 per day. If the company has $75,000 in the bank, how long will it be until the company runs out of money?					
18. The marketing department of Toms River Ventures is catering a party for the purchasing agents of its best customers. 354 people are expected at the party, and 118 bottles of sparkling water have been ordered. How many people does each bottle serve?					

SOLUTIONS

PRACTICE QUESTIONS FOR CHOOSING THE CORRECT OPERATION BEGIN ON PAGE 53.

REINFORCEMENT PROBLEM: CHOOSING THE CORRECT OPERATION

1.

Item	Add.	Sub.	Mult.	Div.	Answer
1. Texarkana Company began the week with $15,404 in its checking account. There is $3,400 in the account at the end of the week. By how much did the account change?		✘			$12,004
2. McLennan Business Supplies sold 125 computers, 1,127 notebooks, 16 printers, 5 fax machines, 10 modems, and 422 pens. How many items did the company sell?	✘				1,705
3. K.C.'s barbecue restaurant cooked 350 meals and used up 25 gallons of barbecue sauce. How many meals per gallon did the company cook?				✘	14 meals per gallon (350 **meals** / 25 **gallons** = 14 meals per gallon)
4. Austin Company is creating a computer software product that it wants to finish in one year. The company has estimated that approximately 34,000 labor hours will be required to complete the project. If one employee works 2,000 hours per year, how many employees will be required for the project?				✘	17 employees (34,000 **hours** / 2,000 **hours** per employee = 17 employees)
5. Cisco Partnership has 12 employees that earn $23.00 per hour. If each employee works 8 hours per day, what is the daily total pay for all employees? What is the weekly pay for a 5-day week?			✘		$2,208 daily total ($23 × 12 = $276. $276 × 8 = $2,208) $11,040 weekly total ($2,208 × 5)
6. Cerritos Enterprises made sales to customers totaling $478,300. If total expenses were $312,500, what was the profit?		✘			$165,800
7. Fullerton Company had advertising expense of $147,000 and Merced Company had advertising expense of $12,250. The expense for Fullerton was how many times that of Merced?				✘	12 times ($12,250 is contained in $147,000 12 times)
8. The net income of Savannah Company was 4 times the net income of Barnesville Company. If Savannah Company earned $81,000, how much did Barnesville earn?				✘	$20,250 ($81,000 / 4 = $20,250)
9. Martinez Enterprises produces 3 times as many items as Cerritos Enterprises. If Cerritos produced 4,200 units last year, how many did Martinez produce?			✘		12,600 (4,200 × 3)
10. Chi paid $24,500 for a new car. This included $1,300 sales tax and $250 shipping. What was the cost of the car itself?		✘			$22,950

SOLUTIONS

PRACTICE QUESTIONS FOR CHOOSING THE CORRECT OPERATION BEGIN ON PAGE 53.

1, *continued*

Item	Add.	Sub.	Mult.	Div.	Answer
11. Shawnee Company has 12 employees. The weekly payroll for all the employees is a total of $10,800. What is the average weekly wage of the employees?				✘	$900 ($10,800 / 12 = $900)
12. Sinclair Company had the following monthly sales in units: October: 3,205 units; November: 2,250 units; December: 2,249 units. Each unit sells for $14. What is the dollar sales for the three months?	✘ (to find the total units)		✘ (to find the total dollar amount)		$107,856 (3,205 + 2,250 + 2,249 = 7,704. 7,704 × $14 = $107,856)
13. On May 1, El Camino Company had 417 units of supplies. During May, the company ordered and received 2,050 more units and used up 1,819 units. How many units were left on May 31?	✘ (to find the total available)	✘ (to find the amount left over after some were used)			648 (417 + 2,050 = 2,467. 2,467 – 1,819 = 648)
14. Manahoy Partnership constructed two buildings. One building cost $105,000 and the other building cost twice as much. What was the cost of both buildings?	✘ (to find the total cost of the two buildings)		✘ (to find the cost of the second building)		$315,000 ($105,000 × 2 = $210,000. $210,000 + $105,000 = $315,000)
15. Manahoy Partnership has two partners that share profits. Partner A received $57,000 which was 3 times as much as Partner B. How much more money did Partner A earn?		✘ (to find the difference between the earnings of the two partners)		✘ (to find how much Partner B earned)	$38,000 ($57,000 / 3 = $19,000. $57,000 – $19,000 = $38,000)
16. Dawson Company purchased $4,500 of office supplies. At the end of the month, an inventory shows that $715 of supplies are still on hand. How much was used up?		✘			$4,500 – $715 = $3,785
17. Passaic Company has a net outflow of cash at the rate of $2,500 per day. If the company has $75,000 in the bank, how long will it be until the company runs out of money?				✘	$75,000 / $2,500 per day = 30 days
18. The marketing department of Toms River Ventures is catering a party for the purchasing agents of its best customers. 354 people are expected at the party, and 118 bottles of sparkling water have been ordered. How many people does each bottle serve?				✘	354 / 118 = 3 people per bottle

▼ *Decimals*

WHAT ARE DECIMALS?

Introduction

Decimals are used to show numbers that are *between zero and one*. These kinds of numbers are important, and occur frequently.

The place values of decimals

We continue using the place-value table to express amounts, but now we expand the table by putting a period mark directly to the right of the “ones” place. This mark is called a **decimal point**. Any number that is located to the left of the decimal point is less than one.

Place-value table with decimals

The place-value table shows the decimal places for numbers less than one.

Each number increases by 10 times the previous number. ←							Each number decreases by 10 times the previous number. →				
	Ten Thousands	Thousands	Hundreds	Tens	Ones	*Decimal Point*	Tenths	Hundredths	Thousandths	Ten Thousandths	
						.					

The numbers to the left of the decimal point are called **whole numbers** and the numbers to the right of the decimal point are called **decimals**.

WHAT ARE DECIMALS? (continued)

How to say numbers that include decimals

- Say the whole number as you normally would.
- Use the word "and" for the decimal point.
- Say the decimal number as if it were also a whole number.
- Then say the name of the place value of the last decimal digit.

Example: "74.915" would be said as: "seventy-four and nine hundred fifteen thousandths."

It is also acceptable to use the word "point" followed by the name of the digits. This is easier when there are many digits to the right of the decimal point.

Example: You could say: "seventy-four *point* nine one five."

The "tenths" place

In the "tenths" place, one is divided into 10 parts. It takes 10 units in the "tenths" place to equal one.

Examples:
- ".1" is read "one tenth" (a pizza is divided into 10 pieces and you receive one piece).
- ".2" is read "two tenths" (a pizza is divided into 10 pieces and you receive two pieces).
- The number "1.4" is read "one and four tenths" (you have one full pizza and four parts out of 10 of another pizza).

The "hundredths" place

In the "hundredths" place, one is divided into 100 parts. It takes 100 units in the "hundredths" place to equal one, and 10 units in the "hundredths" place to equal one tenth.

Examples:
- ".01" is said as "one hundredth" (ten times smaller than .1).
- ".02" is said as "two hundredths" (ten times smaller than .2).
- The number "1.04" is said as "one and four hundredths" (or "one point zero four").
- The number "23.75" is said as "twenty-three and seventy-five hundredths" (or "twenty-three point seven five").

WHAT ARE DECIMALS? (continued)

The "thousandths" place

In the "thousandths" place, the amounts are 10 times smaller than the "hundredths" place. One is divided into one thousand parts.

Examples:
- ".001" is said as "one one thousandth" (ten times smaller than .01).
- ".002" is said as "two thousandths" (ten times smaller than .02).
- The number "1.004" is said as "one and four thousandths."
- The number "23.758" is said as "twenty-three and seven hundred fifty-eight thousandths" (or "twenty-three point seven five eight").

Use of commas

For some reason that no one seems to remember, commas are not used between digits that are to the right of the decimal point. Therefore, a number like 12.31295 is written without a comma.

Zeroes after the last number

When zeroes are placed *after* the last decimal number, the number is unchanged. The zero merely functions as a placeholder, and its use is optional.

Examples:
- .10 is the same as .1
- .100 is the same as .1
- .370 is the same as .37
- .3700 is the same as .37

Decimals are very popular

Decimals are the most common way of expressing numbers. Of course, money amounts are expressed using decimals, by placing a dollar sign ($) to the left of a decimal number.

P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E

PRACTICE

SOLUTIONS FOR EXPRESSING NUMBERS AS DECIMALS BEGIN ON PAGE 61.

REINFORCEMENT PROBLEMS: EXPRESSING NUMBERS AS DECIMALS

1. Identify the place value of the number written in **bold** type.

Number	Place value
a. 21.**3**	
b. 21.0**3**	
c. 21.00**3**	
d. 21.000**3**	

---

2. Say the following numbers. Use both methods of expression if possible.

a. 12.121	
b. 2.002	
c. $.75	
d. .75	
e. 498.10	
f. .333	
g. 3,529	
h. .845	
i. 3,492.07	
j. $3,492.07	
k. .5	
l. .05	
m. .005	

SOLUTIONS

PRACTICE QUESTIONS FOR EXPRESSING NUMBERS AS DECIMALS BEGIN ON PAGE 60.

REINFORCEMENT PROBLEM: EXPRESSING NUMBERS AS DECIMALS

1.

Number	Place value
a. 21.**3**	tenths
b. 21.0**3**	hundredths
c. 21.00**3**	thousandths
d. 21.000**3**	ten thousandths

2.

a. 12.121	“twelve and one hundred twenty-one thousandths” or “twelve point one two one”
b. 2.002	“two and two thousandths” or “two point zero zero two”
c. $.75	“seventy-five cents”
d. .75	“seventy-five hundredths” or “point seven five”
e. 498.10	“four hundred ninety-eight and one tenth” or “four hundred ninety-eight point one”
f. .333	“three hundred thirty-three thousandths” or “point three three three”
g. 3,529	“three thousand five hundred twenty-nine” (not a decimal number)
h. .845	“eight hundred forty-five thousandths” or “point eight four five”
i. 3,492.07	“three thousand four hundred ninety-two and seven hundredths” or “three thousand four hundred ninety-two point zero seven”
j. $3,492.07	“three thousand four hundred ninety-two dollars and seven cents”
k. .5	“five tenths” or “point five”
l. .05	“five hundredths” or “point zero five”
m. .005	“five thousandths” or “point zero zero five”

ROUNDING DECIMALS

Introduction

The procedure for rounding decimals is the same as for any other number.

<table>
<tr><th>Step</th><th colspan="2">Action</th></tr>
<tr><td rowspan="4">1</td><td colspan="2">In the number selected, determine which place-value digit is to be rounded.

Note: This information is often specified in a problem, or by an instructor or supervisor.</td></tr>
<tr><td>If …</td><td>Then …</td></tr>
<tr><td>the digit to the right of the digit to be rounded is 5 or greater,</td><td>add 1 to the digit to be rounded.</td></tr>
<tr><td>the digit to the right of the digit to be rounded is less than 5,</td><td>the digit to be rounded remains unchanged.</td></tr>
<tr><td>2</td><td colspan="2">Delete each digit to the right of the rounded digit.</td></tr>
</table>

Examples

The table below shows examples of rounding decimals.

Number	Round to this place value	Rounded number
.338	hundredths	.34
54.009	hundredths	54.01
.87220	hundredths	.872
14.847	tenths	14.8
39.08	tenths	39.1
4.7285	thousandths	4.729
9.040708	ten thousandths	9.0407
32.783	ones	33

ADDITION WITH DECIMALS

Overview

The addition of decimals is essentially the same procedure that you learned for addition of whole numbers. It is based upon aligning each digit in its correct place-value location.

Procedure for adding decimals

The table below shows you the procedure for adding decimal numbers.

Step	Action
1	Arrange the numbers vertically so that decimal points are exactly aligned.
2	For numbers with fewer digits to the right of the decimal point, insert zeroes to the right of the last digit until all numbers have the same number of digits on the right side of the decimal point.
3	Add the digits beginning with the right place-value column, and work to the left.
4	Align the decimal point in the answer with the decimal points above it.

Example

Add the following numbers: 15, 309.07, 711.3, .09, 10.885, and 53.007

Step	Action	Example
1	Arrange the numbers vertically so that decimal points are exactly aligned. (Notice how the whole number 15 is aligned.)	15. 309.07 711.3 .09 10.885 + 53.007
2	For numbers with fewer digits to the right of the decimal point, insert zeroes after the last digit until all numbers have the same number of digits on the right side of the decimal point.	15.**000** 309.07**0** 711.3**00** .09**0** 10.885 + 53.007

(table continued on next page)

ADDITION WITH DECIMALS (continued)

Step	Action	Example
3	Add the digits, beginning with the right place-value column, and work to the left.	$\begin{array}{r} {}^{1}\ {}^{1}\ {}^{2}\ \ {}^{1}\ \ \ \\ 15.000 \\ 309.070 \\ 711.300 \\ .090 \\ 10.885 \\ +\quad 53.007 \\ \hline 1,099352 \end{array}$
4	Align the decimal point in the answer with the decimal points above it.	$\begin{array}{r} {}^{1}\ {}^{1}\ {}^{2}\ \ {}^{1}\ \ \ \\ 15.000 \\ 309.070 \\ 711.300 \\ .090 \\ 10.885 \\ +\quad 53.007 \\ \hline 1,099.352 \end{array}$ ↑ **Decimal point is correctly aligned**

SUBTRACTION WITH DECIMALS

Overview

Exactly the same procedure is used for subtraction of decimals as for addition, except that in Step 3 you subtract instead of add.

Example

What is the difference between 495.45 and 281.747?

$$\begin{array}{r} {}^{4}\ {}^{14}\ {}^{4}\ {}^{10} \\ 495.450 \\ -\quad 281.747 \\ \hline 213.703 \end{array}$$

MULTIPLICATION WITH DECIMALS

Overview

Multiplication of numbers with decimals is practically the same as multiplication of whole numbers. The essential feature about multiplying numbers with decimals is that you must carefully count the total number of digits to the right of the decimal point.

Example

Multiply 85.73 and 8.4.

Step	Action	Example
1	Multiply the numbers just as if they were whole numbers. Ignore the decimal point.	$\begin{array}{rrrrrr} & & 4 & 5 & 2 & \\ & & \cancel{2} & \cancel{2} & \cancel{1} & \\ & & 8 & 5 & .7 & 3 \\ \times & & & & 8. & 4 \\ \hline & 3 & 4 & 2 & 9 & 2 \\ 6 & 8 & 5 & 8 & 4 & \\ \hline 7 & 2 & 0 & 1 & 3 & 2 \end{array}$
2	Count the total number of digits that are to the right of the decimal point in both factors. Here: 2 in the top factor and 1 in the bottom factor = 3.	$\begin{array}{rrrrrr} & & 4 & 5 & 2 & \\ & & \cancel{2} & \cancel{2} & \cancel{1} & \\ & & 8 & 5. & 7 & 3 \\ \times & & & & 8. & 4 \\ \hline & 3 & 4 & 2 & 9 & 2 \\ 6 & 8 & 5 & 8 & 4 & \\ \hline 7 & 2 & 0 & 1 & 3 & 2 \end{array}$
3	Begin with the right digit of the product, and count to the left the same total places that you obtained in Step 2. Insert the decimal point so that the product has the same number of digits to the right of the decimal point as the total in Step 2.	$\begin{array}{rrrrrr} & & 4 & 5 & 2 & \\ & & \cancel{2} & \cancel{2} & \cancel{1} & \\ & & 8 & 5. & 7 & 3 \\ \times & & & & 8. & 4 \\ \hline & 3 & 4 & 2 & 9 & 2 \\ 6 & 8 & 5 & 8 & 4 & \\ \hline 7 & 2 & 0. & 1 & 3 & 2 \end{array}$ **The decimal point is three places to the left, starting at the right numeral**

MULTIPLICATION WITH DECIMALS (continued)

Adding zeroes in the product

Sometimes extra zeroes have to be added to the product directly adjacent to the right of the decimal point.

> *Example:* If you multiply 2.18 times .035, you will need to count 5 places in the product, because there are 5 numbers to the right of the decimal point in the factors.

$$\begin{array}{rrrrr} & & & {\scriptstyle 4} & \\ & & 2. & 1 & 8 \\ \times & & .0 & 3 & 5 \\ \hline & 1 & 0 & 9 & 0 \\ & 6 & 5 & 4 & \\ \hline .0 & 7 & 6 & 3 & 0 \end{array}$$

You need to add another zero here

DIVISION WITH DECIMALS

Overview

Dividing decimals is essentially the same as dividing whole numbers. However, decimal points in the divisor and dividend may have to be shifted to the right.

DIVISION WITH DECIMALS (continued)

Procedure The following table shows you the procedure for dividing decimals.

Step	Action	Example
1	Determine if the divisor a whole number. • If yes, only do Step 1: divide as you usually would with whole numbers, and be sure the decimal point in the quotient is aligned directly above the decimal point in the dividend. • If no, go to Step 2.	**Divide 2.55 by 75** **Decimal points are aligned** . 0 3 4 7 5 | 2. 5 5 0 2 2 5 3 0 0 3 0 0 **Adding one zero adds one place-value to the quotient**
2	If the divisor is a number with a decimal, make the divisor a whole number. Do this by moving the decimal point in the divisor to the right until the divisor becomes a whole number.	**Divide 32.5 by 1.75** 1 . 7 5 becomes 1 7 5
3	Move the decimal point in the dividend to the right by the same number of places as the decimal point was moved in the divisor. If there are not enough numbers, add zeroes to the dividend. *Note:* You can add extra zeroes beyond this, if you wish to show more places to the right of the decimal point in the quotient. Doing this gives a more precise answer.	3 2 . 5 becomes 3 2 5 0

(table continued on next page)

DIVISION WITH DECIMALS (continued)

Step	Action	Example
4	Be sure the decimal point in the quotient is aligned directly above the decimal point in the dividend. Divide as usual.	**Decimal points are aligned** 18.57 175)3250.00 175 1500 1400 1000 875 1250 1225 **Remainder** 25 **2 extra zeroes are added**

Desired accuracy

In general, the custom is to carry out division one place value beyond the desired place value, and then round the desired place value.

Examples:

- In the example in Step 4 above, we will have accuracy to the tenths place value. The number will be rounded to 18.6.
- If we wanted accuracy to the hundredths place, we would add one more zero to the dividend (3250.000) and carry out the division to the thousandths place. This would result in 18.571, which would be rounded to 18.57.

Zeroes on the left side of the quotient

When the divisor is bigger than the dividend, the quotient that results will be such a small number that you need to place zeroes on the left side of the quotient, immediately after the decimal point. This follows exactly the same rule that you learned when dividing whole numbers.

Rule

You must place a zero above any number in the dividend that is brought down but does not make the partial dividend equal to or greater than the divisor.

DIVISION WITH DECIMALS (continued)

Example

The example below shows the application of the rule with a small decimal quotient.

Action	Example
Determine what is the largest multiplier of the divisor (9) that will not give a product that exceeds the partial dividend of 80. This amount is 8, because 8 times 9 gives a product of 72.	**Divide .801 by 9** $\begin{array}{r} 8 \\ 9\overline{)\,.80\;1} \\ 72 \end{array}$
Because the multiplier of 8 had to be put in the hundredths place, it is necessary to clarify this location by placing a zero in the tenths place.	$\begin{array}{r} .08 \\ 9\overline{)\,.80\;1} \\ \underline{72} \end{array}$
Continue the division in the normal way.	$\begin{array}{r} .089 \\ 9\overline{)\,.801} \\ \underline{72} \\ 81 \\ \underline{81} \\ 0 \end{array}$

Multiplying and dividing by numbers greater than 1 or less than 1

When multiplying and dividing, it is a good idea to know ahead of time whether the result is going to be a bigger number or a smaller number. This is very useful in avoiding mistakes.

Multiplication example

If the multiplier factor is …	then the answer is always …	Example
greater than 1	*bigger* than the other factor	Multiply 50 by 4 (multiplier). Answer: 200
less than 1	*smaller* than the other factor	Multiply 50 by .4 (multiplier). Answer: 20
exactly 1	the same number	Multiply 50 by 1. Answer: 50

DIVISION WITH DECIMALS (continued)

Division example

If the divisor is …	then the answer is always …	Example
greater than 1	*smaller* than the dividend	Divide 200 by 5 (divisor). Answer: 40
less than 1	*bigger* than the dividend	Divide 200 by .5 (divisor). Answer: 400
exactly 1	the same number	Divide 200 by 1 (divisor). Answer: 200

PRACTICE

SOLUTIONS FOR CALCULATING WITH DECIMALS BEGIN ON PAGE 72.

REINFORCEMENT PROBLEMS: CALCULATING WITH DECIMALS

1. **Various calculations.** Complete the following calculations without using a calculator:

Calculation	Answer
a. 718.852 + 224.36 + 141.039	
b. \$731.44 × \$289.03 (round to hundredths)	
c. .8975 ÷ 4 (round to thousandths)	
d. 214.072 – 99.39	
e. .0075 ÷ .05	
f. .08 × .07	
g. \$1,100.98 + \$384.66 + \$903.21 + \$288.17	
h. 108.49 – 17.136	
i. .27075 – .09057	
j. .725 × .083 (round to ten thousandths)	
k. \$10,500 ÷ .4	
l. 75 ÷ 225 (round to thousandths)	
m. 550 ÷ .75 (round to thousandths)	
n. 550 × .75	
o. .55 ÷ 20 (to ten thousandths place)	
p. .09 ÷ 12	
q. 7.85 × 12.42	
r. 29.5 × 21.009	

2. **Estimating results.** In the table below, indicate if the answer will be bigger or smaller:

Multiplication	a. 75 × 8	b. 75 × .8	c. 219 × .37	d. 219 × 3.7
Bigger or smaller than top factor?				

Division	e. \$2,500 / 5	f. \$2,500 / .5	g. 8,315 / .25	h. 8,315 / 2.5
Bigger or smaller than dividend?				

SOLUTIONS

PRACTICE QUESTIONS FOR CALCULATING WITH DECIMALS BEGIN ON PAGE 71.

REINFORCEMENT PROBLEMS: CALCULATING WITH DECIMALS

1.

Calculation	Answer
a. 718.852 + 224.36 + 141.039	1,084.251
b. $731.44 × $289.03 (round to hundredths)	$211,408.10
c. .8975 ÷ 4 (round to thousandths)	.224
d. 214.072 – 99.39	114.682
e. .0075 ÷ .05	.15
f. .08 × .07	.0056
g. $1,100.98 + $384.66 + $903.21 + $288.17	$2,677.02
h. 108.49 – 17.136	91.354
i. .27075 – .09057	.18018
j. .725 × .083 (round to ten thousandths)	.0602
k. $10,500 ÷ .4	$26,250
l. 75 ÷ 225 (round to thousandths)	.333
m. 550 ÷ .75 (round to thousandths)	733.333
n. 550 × .75	412.5
o. .55 ÷ 20 (to ten thousandths place)	.0275
p. .09 ÷ 12	.0075
q. 7.85 × 12.42	97.497
r. 29.5 × 21.009	619.7655

2.

Multiplication	a. 75 × 8	b. 75 × .8	c. 219 × .37	d. 219 × 3.7
Bigger or smaller than top factor?	bigger (600)	smaller (60)	smaller (81.03)	bigger (810.3)

Division	e. $2,500 / 5	f. $2,500 / .5	g. 8,315 / .25	h. 8,315 / 2.5
Bigger or smaller than dividend?	smaller ($500)	bigger ($5,000)	bigger (33,260)	smaller (3,326)

▼ *Percent*

OVERVIEW OF PERCENT

Introduction

A percent is a very common way of expressing the relationship between two numbers. This is probably because when a number is expressed as a percent, the number often seems more natural or easier to understand than when it is expressed in a different way. Percents are used very frequently in business applications such as expressing profits, expenses, tax rates, markups and markdowns, probabilities, and changes in something.

Percent defined

Percent means "per 100" or "units per 100." When using the word "percent," you are saying that you are comparing a number to 100, which is considered a standard reference. The word "percent" derives from the Latin expression *per centum* that means "per 100."

Note: The word "percentage" is often used instead of "percent."

Symbol

The symbol "%" means "percent."

"Rate" and percent

Since a percent is the result of comparing one number to another, percent is really just a particular way of expressing a rate, which you learned when studying division. In fact, the word "rate" is often used to refer to percent. This can be a little confusing. A careful reading of facts will usually clarify what is meant.

The whole amount of anything

Whenever we wish to refer to a whole amount of anything, we call it "100 percent." This serves as a standard reference. Any percent less than 100 percent is less than the reference amount. Any number more than 100 percent is greater than the reference amount.

Why use 100?

There is nothing magical or mysterious about using 100 as a point of reference. We could use any number. However, 100 is used because most people seem to find the round amount of 100 to be a simple and understandable point of reference, so it has become standardized.

OVERVIEW OF PERCENT (continued)

Examples

- To express the idea that 7 out of every 100 units are unsatisfactory, we would say, "We have a rejection rate of 7 percent." (We could also have said "seven hundredths.")
- To say that $28 out of every $100 of income is paid as taxes, we would say that the "tax rate is 28%." (We could also have said "twenty-eight hundredths.")
- To say that Jones Company has $125 of sales for every $100 of sales of Smith Company, we would say, "Jones Company sales are 125% of Smith Company sales." (We could also have said "one hundred twenty-five hundredths.")

CONVERTING NUMBERS TO AND FROM PERCENT

Any number can be 100%

It is not necessary to compare a number to exactly 100 in order to use percent. Any number can be expressed as a percent of any other number which represents a whole amount. Use the procedure below.

Procedure: convert a number into a percent

Imagine that we own a store that sells computer equipment. Suppose that we have 53 computers in our store, which are part of a total of 212 different items for sale in the store. The following table shows how to express 53 as a percent of 212.

Step	Action	Example
1	Identify a **base amount**. The base amount is the entire or whole amount of something, or a reference amount. It represents 100%. *Note:* The base amount often follows the word "of."	The base amount is 212 (units of merchandise).
2	Identify the **portion**. The portion is the number that you are comparing to the base amount.	The portion is 53 (units of merchandise).

(table continued on next page)

CONVERTING NUMBERS TO AND FROM PERCENT (continued)

Step	Action	Example
3	Convert the portion to a decimal number by dividing the base amount into the portion.	53 / 212 = .25 (portion now in hundredths)
4	Convert the decimal to a percentage. Move the decimal point two places to the right by multiplying the decimal by 100.	.25 becomes 25
5	Add a percent symbol after the number.	25% (hundredths now expressed as percent)

Converting a percent to a decimal

If there is a number that is expressed as a percentage which you wish to convert to decimal, reverse the steps:

Step	Action	Examples	
1	Remove the percent symbol.	25% becomes 25	125% becomes 125
2	Move the decimal point two places to the left by dividing the number by 100.	25 / 100 = .25	125 / 100 = 1.25

Caution!

Numbers less than 1% are easy to misread!

When a number is less than 1%—that is, less than one part in a hundred—a decimal point is placed in front of the left digit of the percent. Be careful when reading these numbers. Examples:

	These …	both mean …	and NOT this …
Written as %	.8%	eight tenths of one percent	8% (eight percent)
Written as decimal	.008		

	These …	both mean …	and NOT this …
Written as %	.25%	twenty-five hundredths of one percent	25% (twenty-five percent)
Written as decimal	.0025		

PRACTICE

REINFORCEMENT PROBLEM: CONVERTING TO AND FROM PERCENT

1. **Conversions.** Without using a calculator, convert the percent numbers to decimals and the decimal numbers to percent, and write your answers in the blank spaces:

Percent	Decimal	Percent	Decimal	Decimal	Percent	Decimal	Percent
82%		7.5%		.5		2.49	
8.2%		.75%		.05		12.49	
.82%		750%		.005		.78	
820%		.082%		.2788		.0333	
3.892%		44.175%		.0012		375	
.031%		1,749%		.084		3.75	

SOLUTIONS

REINFORCEMENT PROBLEM: CONVERTING TO AND FROM PERCENT

1.

Percent	Decimal	Percent	Decimal	Decimal	Percent	Decimal	Percent
82%	.82	7.5%	.075	.5	50%	2.49	249%
8.2%	.082	.75%	.0075	.05	5%	12.49	1,249%
.82%	.0082	750%	7.5	.005	.5%	.78	78%
820%	8.2	.082%	.00082	.2788	27.88%	.0333	3.33%
3.892%	.03892	44.175%	.44175	.0012	.12%	375	37,500%
.031%	.00031	1,749%	17.49	.084	8.4%	3.75	375%

ROUNDING PERCENT

Procedure

The procedure for rounding percent is essentially the same as for any other number. The table below shows this procedure with examples.

Step	Action	Example
1	Determine which place-value digit you wish to round.	Round 45.3937% to the hundredths place.
2	• If the digit to the right of the rounded digit is 5 or greater, add 1 to the rounded digit. • If the digit to the right of the rounded digit is less than 5, the rounded remains unchanged.	45.3**9**37%
3	Delete all the digits to the right of the rounded digit.	45.39%

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION

Convert to decimals

If you are given percentages to add, subtract, multiply, or divide, simply convert the percentages to decimals (see page 75). Then perform the operations in the same way that you learned about calculating with decimals.

Caution!

Confusing expressions!

Do these expressions have the same meaning?

- "What is 20% of ten?"
- "Ten is 20% of what?"

Answer: Different meanings!

Here are the correct calculations:

- "What is 20% of ten?" means 10 × .2 = 2
- "Ten is 20% of what?" means 10 ÷ .2 = 50

Guideline

- A **"percent of" a number** means to multiply to obtain a portion of that number.
- A **number that is a "percent of"** ***another number*** means divide to find the other number.

ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION (continued)

Quick practice

Calculate and round decimal answers to nearest hundredth.

Question	Answer
What is 25% of 125?	31.25
125 is 25% of what number?	500
80% is 20% of what number?	4
What is 20% of 80%?	.16
What is 120% of 120?	144
120 is 120% of what number?	100

USING PERCENT TO SOLVE PROBLEMS

Overview

Percent calculations are extremely useful, and have a lot of business applications. In general, percent is used to solve problems in the following four areas:

- the three basic types of percent calculation
- showing changes
- finding a base or a portion when they are combined
- comparing different bases

Each of these topics is discussed below.

THREE BASIC TYPES OF PERCENT CALCULATION

Introduction

When you are working in business situations, you will encounter three basic types of percent calculations which repeatedly occur. It is important that you become familiar and comfortable with these three calculations. The calculations involve three related items: base, portion and rate.

Definition: "base"

A "base" is a reference amount to which another number is compared or is in some way related.

THREE BASIC TYPES OF PERCENT CALCULATION (continued)

Definition: "portion"

A "portion" is the number that is compared to or related to the base amount. The portion is usually expressed as some percent of the base.

Definition: "rate"

In base/portion/rate problems, the word "rate" means percent.

> *Note:* This is a slightly different use of the word "rate" than when it is used to compare one amount to another as a "per unit" description."

Three basic types of percent calculations

These are three basic types of percent calculations:

- **Type 1: Finding the base amount**
You know the percent and you know the portion, but what was the base number?

> *Example:* Your business collected $45,000 in sales tax. You know that sales tax is 5% of total sales. What was the total sales? (Here, $45,000 is the portion and 5% is the rate.)

- **Type 2: Finding the portion**
You know the rate (percent) and you know the base, but what was the portion?

> *Example:* Your business had $900,000 of sales. The sales tax is 5% of all sales. How much sales tax should you have collected? (Here, $900,000 is the base and 5% is the rate.)

> *Note:* Usually the portion is smaller than the base, but this is not always true. The portion can also be greater than the base, if the percent is greater than 100%.

- **Type 3: Finding the rate**
The percent amount is also called the **rate**. In this type of problem, you know the base and you know the portion, but you need to find the percent. This calculation is exactly the same as what you have already done to convert a number to a percent.

> *Example:* If $45,000 of sales tax was collected and total taxable sales was $900,000, what was the rate of tax (the percent of sales)? (Here, $45,000 is the portion and $900,000 is the base.)

THREE BASIC TYPES OF PERCENT CALCULATION (continued)

Memory aid

Use the diagram below as a memory aid when doing calculations to solve the above three types of problems:

Portion	
Rate	**Base**

How to use the memory aid diagram

1. Identify which one of the three elements you need to find. Put your finger over the box that contains this element.

2. Use the other two remaining elements to solve the problem:

- If the two remaining elements are side by side, then multiply them.
- If one remaining element is above the other remaining element, then divide the lower element into the one above.

How to identify the portion, rate and base

Now that you know the three types of percent calculations, the next step is learning to identify these amounts in problems. An easy way to do this is to use the diagram above but change the words so it looks like this:

Some Number =	
% of	Another Number

Let's try this on examples from the previous page:

Type 1 Example:

$**45,000** =	
5% of	Another Number

Putting your finger over "Another Number" (the base) means that you divide $45,000 by .05 to find the number (the base.) Answer: $900,000.

Type 2 Example:

Some Number =	
5% of	$**900,000**

Putting your finger over "Some Number" (portion) means that you multiply .05 times $900,000 to find the number (portion). Answer: $45,000.

PRACTICE

REINFORCEMENT PROBLEM: IDENTIFY THE PORTION, BASE, AND RATE

1. This problem does not require any calculations. What is required is that you correctly *identify* the portion, base, and rate in each problem description. In the table below, write the amount of the portion, base, or rate for each separate situation. If one of these items is what must be calculated, write a check mark (✓) for the item that must be calculated. The first problem is an example.

Problem Description	Portion	Base	Rate
a. If 1,200 pink computers were sold, and this is 10% of all computer sales, how many computers were sold?	1,200	✓	10%
b. Andrea made a 20% down payment on a house. She paid $55,000. What did the house cost?			
c. Last year Kelly Company had sales of $485,000. The company forecasts a 7% increase in sales. How much will the increase be?			
d. Anandi Company had sales of $514,000 and a net income of $74,530. What percent is net income of sales?			
e. Jay earns $40 per hour and John earns $50 per hour. John's earnings are what percent of Jay's earnings?			
f. Diem Art Gallery paid an 8% commission when it sold a painting for $15,000. How much was the commission?			
g. Hobbs Enterprises purchased $75,000 of new equipment, which was 20% of the amount that had been budgeted for equipment. How much had been budgeted?			
h. .35 is 15% of what amount?			
i. What is 15% of .35?			

SOLUTIONS

REINFORCEMENT PROBLEM: IDENTIFY THE PORTION, BASE, AND RATE

1.

Problem Description	Portion	Base	Rate
a. If 1,200 pink computers were sold, and this is 10% of all computer sales, how many computers were sold?	1,200	✓	10%
b. Andrea made a 20% down payment on a house. She paid $55,000. What did the house cost?	$55,000	✓	20%
c. Last year Kelly Company had sales of $485,000. The company forecasts a 7% increase in sales. How much will the increase be?	✓	$485,000	7%
d. Anandi Company had sales of $514,000 and a net income of $74,530. What percent is net income of sales?	$74,530	$514,000	✓
e. Jay earns $40 per hour and John earns $50 per hour. John's earnings are what percent of Jay's earnings?	$50	$40	✓
f. Diem Art Gallery paid an 8% commission when it sold a painting for $15,000. How much was the commission?	✓	$15,000	8%
g. Hobbs Enterprises purchased $75,000 of new equipment, which was 20% of the amount it had been budgeted for equipment. How much had been budgeted?	$75,000	✓	20%
h. .35 is 15% of what amount?	.35	✓	15%
i. What is 15% of .35?	✓	.35	15%

EXAMPLES OF EACH TYPE OF CALCULATION

Finding the base

Your business collected $45,000 in sales tax. You know that sales tax is 5% of total sales. What was the amount of total sales?

Portion (some number =)	
Rate (% of)	**Base** (another number)

We are asking: "$45,000 is 5% of what number?" The number we are trying to find is the base, so put your finger over the base (dark rectangle): $45,000 / .05 = $900,000 of sales.

Your travel expense for the year totaled $83,500. The accounting department tells you that your travel expense was 21.8% of the total travel expense for the entire business. How much was the total travel expense?

Portion (some number =)	
Rate (% of)	**Base** (another number)

We are asking: "$83,500 is 21.8% of what number?" The number we are trying to find is the base, so put your finger over the base (dark rectangle). Following the procedure, divide the rate into the portion: $83,500 / .218 = $383,027.52 total travel expense.

Finding the portion

Your business had $900,000 of sales. The sales tax is 5% of all sales. How much sales tax should you have collected?

Portion (some number =)	
Rate (% of)	**Base** (another number)

We are asking: "What number is 5% of $900,000?" The number that we are trying to find is the portion, so put your finger over the portion (dark rectangle). Following the procedure, multiply the base by the rate (percent): $900,000 × .05 = $45,000.

EXAMPLES OF EACH TYPE OF CALCULATION (continued)

Finding the portion (continued)

Acme Company has budgeted the product development expense as 12% of total expenses. If total expenses are budgeted at $715,000, how much will be budgeted for product development?

Portion (some number =)	
Rate (% of)	**Base** (another number)

The reference (base) amount is $715,000. The rate is 12%. We are asking: "What number is 12% of $715,000?" The number that we are trying to find is the portion, so put your finger over the portion (dark rectangle). Following the procedure, multiply the base by the rate (percent): $715,000 × .12 = $85,800.

The portion can be bigger than the base: total utilities expense this month is $10,000. If we budget utilities expense for next month at 110% of this month, how much is budgeted for next month?

Portion (some number =)	
Rate (% of)	**Base** (another number)

The reference (base) amount is $10,000. We are asking: "What number is 110% of $10,000?" The number that we are trying to find is the portion, so put your finger over the portion (dark rectangle). Following the procedure, multiply the base by the rate (percent): $10,000 × 1.10 = $11,000.

Finding the rate

Finding the rate is simply converting a number to a percent, which was presented to you on a previous page. Here is an additional example:

If $45,000 of sales tax was collected and total taxable sales was $900,000, what was the rate of tax (the percent)?

Portion (some number =)	
Rate (% of)	**Base** (another number)

We are asking: "What percent is $45,000 of $900,000?" The number that we are trying to find is the rate, so put your finger over the rate (dark rectangle). Following the procedure, divide the portion by the base: $45,000 / $900,000 = .05 = 5%.

EXAMPLES OF EACH TYPE OF CALCULATION (continued)

Caution!

Converting to %

Notice that in the above calculation, the .05 was converted to a percent format of 5%. The procedure to convert the decimal to a percent was: .05 × 100 = 5%.

Multiplying by 100 moves the decimal point two places to the right, which results in the number 5. Only *after* you multiply by 100 do you add the % symbol.

Note: The answer is *NOT* .05%; you cannot just add a % symbol!

PERCENT CALCULATIONS FOR SHOWING CHANGES

Overview of change situations

There are two common situations involving *change* that call for the use of percent:

- express a change as a percent of a base amount
- calculate a new base amount

Situation #1: express change as a percent

People in business are always concerned about change. They frequently want to express change in percentage terms, rather than using only dollar amounts. This is because percent is easy to understand, and often comparison is more meaningful when percent is used instead of dollars.

Example

Company A had a $15,000 increase in sales during 2008. Company B had a $20,000 increase in sales in 2008. Which increase is better? Of course, $20,000 is greater than $15,000, but suppose I give you additional information: Company A sales in the prior year were $50,000 and Company B sales in the prior year were $500,000. Does this data change your opinion?

To clarify the situation, you can express the change as a percent of the base amount in the prior year.

PERCENT CALCULATIONS FOR SHOWING CHANGES (continued)

Calculating the percent change

In this situation, *the* ***change*** *represents a* ***portion*** *of a base amount*. We are going to express the change as a percent rate.

Portion	
Rate	**Base**

Procedure

Using the information in the example, the following table shows you how to calculate the rate of change.

Step	Action	Examples	
		Company A	**Company B**
1	Identify the base amount (the year we are comparing to).	$50,000	$500,000
2	Identify or calculate the portion.	$15,000	$20,000
3	Divide the portion by the base.	$15,000 / $50,000 = .30 = 30%	$20,000 / $500,000 = .04 = 4%

What a difference! Company A sales grew at the rate of 30% per year. Company B sales grew at the rate of only 4%.

An example with all percent

The marketing department’s share of the total budget decreased from 30% to 18% of the total budget. What percent decrease does this change represent?

Step	Action	Example
1	Identify the base amount.	30%
2	Identify or calculate the portion.	30% – 18% = 12%
3	Divide the portion by the base.	.12 / .3 = .4 or a 40% decrease

PERCENT CALCULATIONS FOR SHOWING CHANGES (continued)

Interpreting the answer

This problem is a little tricky because the base (30%) and the change portion (12%) represent a share of the total (which we are accustomed to seeing in dollars). In other words, this problem is about a percent change in a percent, not dollars.

Situation #2: calculate a new base

In this situation, you are given the base amount. You are also given the percent of increase or decrease in the base amount. You have to calculate what the new total base amount will be by adding the amount of the increase or subtracting the amount of the decrease.

Increases in the base

There are two ways to calculate the increase in base kind of problem. The second way is faster.

> *Example:* You are earning $3,800 per month. Because of your good work, you receive an 8% pay increase. What is the amount per month that you will be earning after the increase?

METHOD 1		
Step	**Action**	**Example**
1	Identify the base amount.	$3,800
2	Identify the percent change.	8% (increase)
3	Multiply the base by the percent to get **the amount of change.**	$3,800 × .08 = $304 (increase)
4	Add the increase to the old base to find the new base.	$3,800 + $304 = $4,104 *Warning:* It is NOT CORRECT to simply add 8 to $3,800 and get the result of $3,808.

METHOD 2 (Better)		
Step	**Action**	**Example**
1	Identify the base amount.	$3,800
2	Add the % increase to 100% and express as a decimal.	100% + 8% = 108% = 1.08
3	Multiply the old base by the decimal to get the new base.	$3,800 × 1.08 = $4,104

PERCENT CALCULATIONS FOR SHOWING CHANGES (continued)

Decreases in the base

There are two ways to solve the decrease in base kind of problem. The second way is faster.

Example: The supplies expense budget this year for your department is $8,500. You are told that next year this budget will be reduced by 15%. What will your supplies budget be for next year?

METHOD 1		
Step	**Action**	**Example**
1	Identify the base amount.	$8,500
2	Identify the percent change.	15% (decrease)
3	Multiply the base by the percent to get the amount of decrease.	$8,500 × .15 = $1,275 (decrease)
4	Subtract the decrease from the old base to find the new base.	$8,500 – $1,275 = $7,225

METHOD 2 (Better)		
Step	**Action**	**Example**
1	Identify the base amount.	$8,500
2	Subtract the % decrease from 100% and express as a decimal.	100% – 15% = 85% = .85
3	Multiply the old base by the decimal to get the new base.	$8,500 × .85 = $7,225

HOW TO FIND A BASE OR PORTION WHEN THEY ARE COMBINED

Introduction

Sometimes an unknown base and an unknown portion are combined together into one total. If you know what percent the portion is of the base, then you can calculate either the base or the portion, or both.

HOW TO FIND A BASE OR PORTION WHEN THEY ARE COMBINED (continued)

Example

You own a bookstore that calculates sales tax at the end of every month. At the end of June, total receipts including sales tax are $270,000. The sales tax rate is 8%. What is the amount of the sales tax? What is the amount of sales?

In this example, the base (the sales) is combined with the portion (the sales tax) which is 8% of the base. Together, the base and portion are $270,000.

Procedure

Follow the procedure in the table below to find a base or portion when both are combined into one total.

Step	Action	Example
1	Add the rate to 100% and convert to a decimal.	100% + 8% = 108% = 1.08
2	Find the base first by dividing the total by the decimal from Step 1. *Note:* It is important to see that the base always represents 100%. Together, the portion and the base are 100% plus whatever additional percent of the base the portion is (in this case, another 8% of the base). In other words, $270,000 is 108% of the base.	$270,000 / 1.08 = $250,000 The amount of sales excluding tax is $250,000
3	To find the portion, subtract the base from the total.	$270,000 – $250,000 = $20,000 (sales tax)

HOW TO FIND A BASE OR PORTION WHEN THEY ARE COMBINED (continued)

Caution

It may be tempting to simply multiply 8% times $270,000 to find the sales tax, but this will not work. Why? Because the $270,000 *already includes sales tax*. The tax should only be calculated on the actual sales (the true base). If you multiply $270,000 by 8%, you would be calculating sales tax on *both* the sales and sales tax! Your answer would overstate the sales tax.

Another example

This year, the attendance at the baseball world series was 885,320 people. This was a 9% increase from last year. How many people attended last year? How many more people attended this year than last year?

Solution:
885,320 / 1.09 = 812,220 (approximately) attended last year.
885,320 – 812,220 = a 73,100 increase.

Example with decrease

In the example above, suppose that the 885,320 attendance this year was a 9% decrease from last year. How many people attended last year? How many fewer people attended this year than last year?

Solution:
885,320 / .91 = 972,879 (approximately) attended last year.
972,879 – 885,320 = an 87,559 decrease.

COMPARISON WITH DIFFERENT BASES

Overview

In business, it is frequently necessary to compare portions that are related to different bases. Calculating a percent for the comparison provides a more useful comparison.

COMPARISON WITH DIFFERENT BASES (continued)

Example

Mega Company had $985,000 of sales last year. Its total operating expenses were $384,150. Mini Company had sales of $91,000 and its operating expenses were $37,310. Which company is operating more efficiently?

It is very difficult to compare the companies by just looking at the dollar amount of operating expenses. Clearly Mega Company had more expenses, but it is also a much bigger company.

Solution:
Compare the expenses (portions) as percentages of the base amounts (sales). Comparing percents, we see that Mega is operating more efficiently:

- Mega Company expenses: $384,150 / $985,000 = .39 = 39% of sales
- Mini Company expenses: $37,310 / $91,000 = .41 = 41% of sales

PRACTICE

SOLUTIONS FOR PERCENT CALCULATIONS BEGIN ON PAGE 95.

REINFORCEMENT PROBLEMS: PERCENT CALCULATIONS

1. **Calculation with decimals, converting decimals and percents.** Convert the following percents to decimals and then do the indicated calculations. Show your answers as both decimals and percents.

Calculate …	Decimal Answer	Percent Answer
37% × 54%		
37% × 5.4%		
37% × .54%		
73.07% + 3.215%		
.95% + .85%		
143% + 100%		
35% ÷ 25%		
35% ÷ 2.5%		
35% ÷ .25%		
35% ÷ 250%		
99.9% – 82.3%		
152.7% – 42.875%		
.7% – .385%		

2. **Analyze common expressions.** Answer the following questions:

Question	Answer	Portion	Rate	Base
a. What number is 115% of 80?				
b. What number is 25% of 350?				
c. What number is 20% of 80%?				
d. $7,500 is 80% of what number?				
e. 350 is 25% of what number?				
f. 35% is 50% of what number?				
g. .20% × 80% is what number?				
h. .75% × 150 is what number?				
i. .9% × .8% is what number?				

P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E ▪ P R A C T I C E

PRACTICE

SOLUTIONS FOR PERCENT CALCULATIONS BEGIN ON PAGE 95.

3. **Identify the base, the portion, and the rate.** Before you can do a correct calculation involving a base, a portion, and a rate, you must first be able to identify them! In this problem, we do not care about doing calculations. *Without doing any calculations,* read each situation, then identify the base, the portion, and the rate by writing the amount of the item in the correct column. If the item is the amount we are trying to calculate, then place a check mark (✓) in the correct column. Use the first situation as an example. (If you want to do the calculations, answers are provided.)

Situation	Base	Portion	Rate
a. Anne purchased a new car for $20,000 and she made a 25% down payment. What was her down payment?	$20,000	✓	25%
b. Last year, the revenue for Acme Company was $820,000. This year, the revenue increased by $98,400. What was the percent increase?			
c. At year end, the price of the stock of Jain Company was $24. This amount was 115% of last year's price. What was the price last year?			
d. O'Leary's Coffee Shoppe surveyed its customers. Of the 500 customers surveyed, 215 preferred decaffeinated brew instead of regular brew. What percent of customers want decaffeinated brew?			
e. Dennis just purchased a new truck, which the salesman claimed will have 5% better gas mileage than Dennis' old truck. The old truck got 14 miles per gallon. What *improvement* in mileage should Dennis expect?			
f. So far this year, Rapacious Corporation has spent $3,750 for supplies, which is 85% of its total supplies budget. What is the total supplies budget?			
g. Smilin' Norm Toy receives a 12% commission of the price of each used car that he sells. This month, Norm received $4,300 in commissions. What is the total price of all the cars that he sold this month?			
h. Passionate Poster Company sells three types of posters: small (15% of sales), medium (72%), and large (13%). The company sold 10,000 posters last month. How many were small posters?			
i. In the A, B, C, Partnership, partner A's share of profits just decreased from 60% to 45%. What was the percent decrease in A's share?			
j. Maxwell Corporation now has a 32% market share, which is 91% of what it was last year. What was the percent market share last year?			

PRACTICE

SOLUTIONS FOR PERCENT CALCULATIONS BEGIN ON PAGE 95.

4. **Answer typical business questions that involve percent.** Calculate the answer to each of the independent situations below. Show your calculation and answer in good form under each question.

   a. At the California state fair, Bjork's hot dog stand began the day with 12 gallons of mustard. At the end of the day, 75% of the mustard is gone. How many gallons of mustard were used?

   b. At the state fair, Chang's ice cream booth began the day with 200 gallons of rocky road ice cream, and at the end of the day, 45 gallons were still unsold. What percent is unsold?

   c. At the state fair, Dorrance's Cotton Candy Concession has 15 pounds of sugar still unused when the day is over. This is 12% of the amount that was available in the morning. How much was available in the morning?

   d. At the state fair, Porter's Fine Flower Shoppe recorded $13,330 of cash collections, which included both sales and sales tax. If the sales tax rate is 7.5% and all sales are taxable, what is the dollar amount of sales tax?

   e. Rochelle purchased 100 shares of stock in a corporation for $35 per share. After a year, the total value of her investment was $3,920. What was the percent change in her investment for the year? (*Note:* the annual percent change in an investment is called annual **rate of return** or **yield**.)

   f. Miller's Hardware is having a sale. Paint is selling for 20% off the regular price. If the regular price of the paint you want is $24 per can, how much is the paint selling for now? Use the faster method to calculate your answer.

   g. (1) If the price of merchandise changed from $200 to $150, what was the percent change?

      (2) If the price of merchandise changed from $150 to $100, what was the percent change?

      (3) If the price of merchandise changed from $100 to $50, what was the percent change?

      The amount of each decrease was $50. But what is happening to the size of the percent change? Why?

   h. The net income of Kona Company is 15% of the total revenue. If net income is $300,000, what is the total revenue?

   i. Lansdale Company had net income of $200,000 and Lehigh Company had net income of $150,000. The Lansdale net income is what percent of Lehigh net income? The Lehigh net income is what percent of Lansdale net income?

PRACTICE

SOLUTIONS FOR PERCENT CALCULATIONS BEGIN ON PAGE 95.

4, *continued*

j. Lackawanna Enterprises had $700,000 of sales revenue this year. Last year, the sales revenue was $350,000. What is the percent change from last year to this year?

k. Last year, your budget was 20% of the total budget. This year it is 15%. What percent did your budget decrease?

l. You have just been hired to work for the Summerdale Grocery Store. You are told that the coffee was marked up by 15% last month, and the manager wants it marked back down to its original price. The current price, which includes the markup, is $2.53 per pound. What price does the manager want?

m. Green Engineering Company had an error rate of 14% when doing tolerance calculations. 82% of these errors occurred when staff worked longer than 10 hours per day. What percent of all tolerance calculations will result in errors related to 10-hour days?

n. The Smith and Jones partnership allocates profits and losses 70% to Smith and 30% to Jones. If Baker enters the partnership and receives a 25% share of profits and losses,

(1) what total percent share of the profits and losses do Smith and Jones receive together?

(2) what percent share will Smith now receive? What percent share will Jones receive?

o. Merchandise is marked up from a cost of $1 to $1.45. What is the markup percentage based on cost? What is the markup percentage based on selling price?

p. **Challenging problem.** The Internal Revenue Service allows small businesses to deduct as an allowable business expense the payments made to certain retirement plans. The retirement deduction is limited to 15% of the net earnings of a business. However, the tax law, tricky as usual, states that the "net earnings" limit means what the net earnings would be after the deduction is subtracted. So, you don't know the amount of net earnings for purposes of the calculation unless you know the amount of the deduction. But you don't know the amount of the deduction unless you know the net earnings!

(1) Suppose a business has $20,000 of "net earnings" before considering the deduction. What amount of net earnings can the 15% deduction be calculated on? What is the amount of the deduction? (*Hint:* View the net earnings before the deduction as some percent greater than what it would be after the deduction.)

(2) What is the true deduction percent limit?

SOLUTIONS

PRACTICE QUESTIONS FOR PERCENT CALCULATIONS BEGIN ON PAGE 91.

REINFORCEMENT PROBLEMS: PERCENT CALCULATIONS

1.

Calculate …	Decimal Answer	Percent Answer
37% × 54%	.1998	19.98%
37% × 5.4%	.01998	1.998%
37% × .54%	.001998	.1998%
73.07% + 3.215%	.76285	76.285%
.95% + .85%	.018	1.8%
143% + 100%	2.43	243%
35% ÷ 25%	1.4	140%
35% ÷ 2.5%	14.	1,400%
35% ÷ .25%	140	14,000%
35% ÷ 250%	.14	14%
99.9% – 82.3%	.176	17.6%
152.7% – 42.875%	1.09825	109.825%
.7% – .385%	.00315	.315%

2.

Question	Answer	Portion	Rate	Base
a. What number is 115% of 80?	1.15 × 80 = 92	92	115%	80
b. What number is 25% of 350?	.25 × 350 = 87.5	87.5	25%	350
c. What number is 20% of 80%?	.2 × .8 = .16	.16	20%	80%
d. $7,500 is 80% of what number?	$7,500 / .8 = $9,375	$7,500	80%	$9,375
e. 350 is 25% of what number?	350 / .25 = 1,400	350	25%	1,400
f. 35% is 50% of what number?	.35 / .5 = .7 (or 70%)	35%	50%	.7
g. .20% × 80% is what number?	.0016 (or .16%)	.0016	.20%	80%
h. .75% × 150 is what number?	1.125	1.125	.75%	150
i. .9% × .8% is what number?	.000072 (or .0072%)	.000072	.9%	.8%

3. (Decimal answers rounded to hundredths place)

Situation	Base	Portion	Rate
a. Anne purchased a new car for $20,000 and she made a 25% down payment. What was her down payment?	$20,000	✓ ($5,000)	25%
b. In the year 2000, the revenue for Acme Company was $820,000. In 2001, the revenue increased by $98,400. What was the percent increase?	$820,000	$98,400 (12%)	✓
c. At year end, the price of the stock of Jain Company was $24. This amount was 115% of last year's price. What was the price last year?	✓ ($20.87)	$22	115%
d. O'Leary's Coffee Shoppe surveyed its customers. Of the 500 customers surveyed, 215 preferred decaffeinated brew instead of regular brew. What percent of customers want decaffeinated brew?	500	215	✓ (43%)

SOLUTIONS

PRACTICE QUESTIONS FOR PERCENT CALCULATIONS BEGIN ON PAGE 91.

3, *continued*

	Situation	Base	Portion	Rate
e.	Dennis just purchased a new truck, which the salesman claimed will have 5% better gas mileage than Dennis' old truck. The old truck got 14 miles per gallon. What *improvement* in mileage should Dennis expect?	14	✓ (.7 mpg)	5%
f.	So far this year, Rapacious Corporation has spent $3,750 for supplies, which is 85% of its total supplies budget. What is the total supplies budget?	✓ ($4,411.77)	$3,750	85%
g.	Smilin' Norm Toy receives a 12% commission of the price of each used car that he sells. This month, Norm received $4,300 in commissions. What is the total price of all the cars that he sold this month?	✓ ($35,833.33)	$4,800	12%
h.	Passionate Poster Company sells three types of posters: small (15% of sales), medium (72%), and large (13%). The company sold 10,000 posters last month. How many were small posters?	10,000	✓ (1,500)	15%
i.	In the A, B, C, Partnership, partner A's share of profits just decreased from 60% to 45%. What was the percent decrease in A's share?	63%	55%	✓ (25%)
j.	Maxwell Corporation now has a 32% market share, which is 91% of what it was last year. What was the percent market share last year?	(35.16%)	32%	91%

4. a. You are calculating the portion: 12 gallons × .75 = 9 gallons used
   b. You are calculating the rate: 45 gallons / 200 gallons = .225 = 22.5%
   c. You are calculating the base: 15 pounds / .12 = 125 pounds
   d. Combined base and portion; you want the portion: $13,330 / 1.075 = $12,400 $13,330 – 12,400 = $930 tax
   e. This is a change problem: $3,920 – $3,500 = $420 increase. $420 portion / $3,500 base = .12 = 12% rate of change
   f. You are calculating the new base: $24 × .8 = $19.20 per can
   g. (1) 50 / 200 = .25 (or a 25% change)
      (2) 50 / 150 = .333 (or a 33.3% change)
      (3) 50 / 100 = .5 = 50% change. The portion remains the same each time (50), but the base is decreasing. This makes the percent change greater.
   h. You are calculating the base: $300,000 / .15 = $2,000,000
   i. Lansdale is approximately 133% ($200,000 / $150,000) of Lehigh. Lehigh is 75% ($150,000 / $200,000) of Lansdale.
   j. This is a change problem: $350,000 (increase amount) / $350,000 (base) = 1 = 100% (100% increase)
   k. This is a change problem: .05 share change (portion) / .20 (base) = .25 = 25% rate of change.
   l. Combined base and portion, you need to know the base: $2.53 / 1.15 = $2.20
   m. .14 × .82 = .1148, or 11.48%.
   n. (1) If the new partner receives 25%, then Smith and Jones together receive 75% of partnership profits and losses.
      (2) Smith will receive .75 × .7 = .525 which is 52.5%. Jones will receive .75 × .3 = .225 which is 22.5%.
   o. This is a change problem: If cost is the base, then the rate is $.45 portion / $1.00 = .45 = 45% markup. If the selling price is the base, then $.45 portion / $1.45 = approximately .31 = 31%.
   p. (1) You need to see that "net earnings" without the deduction is going to be 115% of what it would be after the deduction is subtracted, so this is a combined base and portion problem. $20,000 / 1.15 = $17,391 (base). The deduction is $17,391 × .15 = approximately $2,609 (portion).
      (2) The true overall deduction percent is not really 15%. It is 1 / 1.15 = .86957, so .15 × .86957 = .13044, which is about 13.044%.

Positive and Negative Numbers

OPPOSITES

Definition

Opposites are things that offset each other, cancel each other out, or go in opposite directions. Some things in the world are natural opposites.

Examples of opposites

- In a business, earning income and paying an expense are opposites, and have opposite effects on wealth of the business.
- In an election, a "yes" vote and a "no" vote have opposite effects on the outcome of a proposition.
- In an aircraft, climbing and descending have opposite effects on the altitude.
- Hours of daylight and hours of sunlight are opposites that have opposite effects on temperature.

Common feature of opposites

What all these things have in common is that the force of one opposing thing offsets or cancels out the effects caused by the other.

- Expenses offset the effects of revenue you earned; also, the more revenue you earn, the more the effects of expenses are offset.
- The more "no" votes there are, the effect of "yes" votes are offset; also, the more "yes" votes there are, the more the "no" votes are offset.

IDENTIFYING OPPOSITES

Overview

Because there are so many natural opposite things in business as well as in other activities, it is important to identify opposites and to measure them. When this is done, all the normal rules for mathematics will apply. That means that we can include opposites in any of our calculations.

"Positive" and "negative"

Opposites are usually described by the words "positive" and "negative."

- **"Positive"** means an amount greater than zero. This describes many conditions and situations such as wealth, progress towards a goal, things of substance, movement in a desired direction, and so on.
- **"Negative"** means an amount less than zero.

Note: The opposite of zero is zero.

IDENTIFYING OPPOSITES (continued)

Showing positive amounts

A positive amount is indicated either by a plus sign (+) or by a numeral.

> *Example:* To indicate a positive 5, you can write "+5" or "5." This would be interpreted by the words "positive five" or "five."

Showing negative amounts

A negative amount is indicated by writing a negative sign (–) in front of a numeral.

> *Example:* "–5" would be interpreted by the words "negative five" or "show the opposite of five."

Different meanings for a "–" sign!

A "–" sign (often called a "minus sign") can be used to indicate:

- a negative number, or
- show the opposite of a number, or
- the operation of subtraction

IDENTIFYING OPPOSITES (continued)

Caution!

Interpreting minus signs

It is extremely easy to confuse the different meanings of a "–" sign. Here is how to distinguish what is intended:

- **When the symbol "–" is followed by a numerical amount, but there is no amount in front of the "–" symbol:** this indicates the negative of the number. It also means that you are to show the opposite of a number. The result is the same in either case.

  Example: "–5" indicates the amount of a negative 5. A –5 can also be interpreted as "show the opposite of 5," which is a negative 5.

  Example: "– (–5)" indicates the amount of the negative of negative 5, which is 5. This can also be interpreted as "show the opposite of –5," which is 5.

- **When the symbol "–" is between two amounts:** this indicates subtraction.

  Example: "8 – 5" means subtract 5 from 8.
  Example: "8 – (–5)" means subtract a negative 5 from 8.

Notice how the negative 5 was placed in parentheses "( )" to prevent confusion with the subtraction sign. We will have more to say about subtraction later.

Synonym

A synonym for positive and negative numbers is "signed numbers."

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PRACTICE

REINFORCEMENT PROBLEM: INTERPRETING THE MEANING OF + AND – SIGNS

1. For each of the expressions shown below, briefly interpret the meaning of the expression. Use the first item as an example.

Expression	Explanation
–5	The negative of 5 (or simply "negative five")
– (–3)	
10	
10 – 8	
–9	
7 – (–3)	
7 + (–3)	

SOLUTIONS

REINFORCEMENT PROBLEM: INTERPRETING THE MEANING OF + AND – SIGNS

1.

Expression	Explanation
–5	The negative of 5 (or simply "negative five")
– (–3)	The negative of negative 3 (the opposite of negative 3)
10	Positive 10 (or simply "ten")
10 – 8	Ten minus eight
–9	The negative of nine (or simply "negative nine")
7 – (–3)	Seven minus a negative three
7 + (–3)	Seven plus a negative three

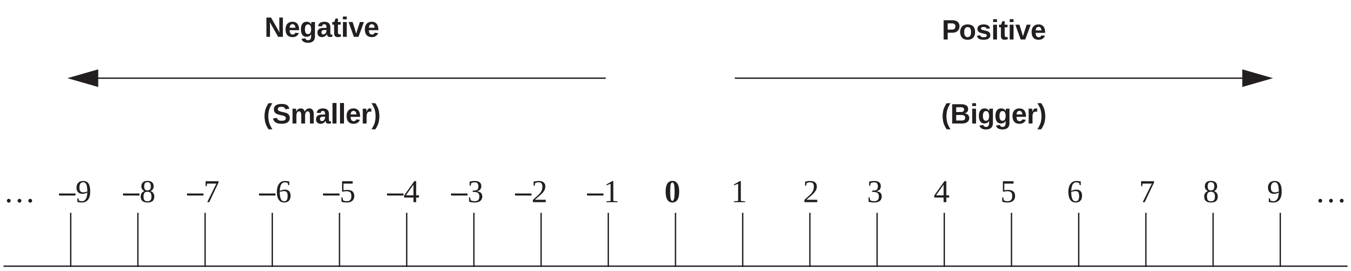
MEASURING OPPOSITES

Overview

When something can be quantified in numbers, we can show the quantity of that thing using positive and negative amounts on a number line.

Number line

The example below shows a number line, in units of 1.



Comparing the numbers

Examples:

- 4 is more than 1, because 4 is to the right of 1.
- 1 is more than –2, because 1 is to the right of –2.
- –2 is more than –5 because –2 is to the right of –5.
- –5 is less than 5, because –5 is to the left of 5.

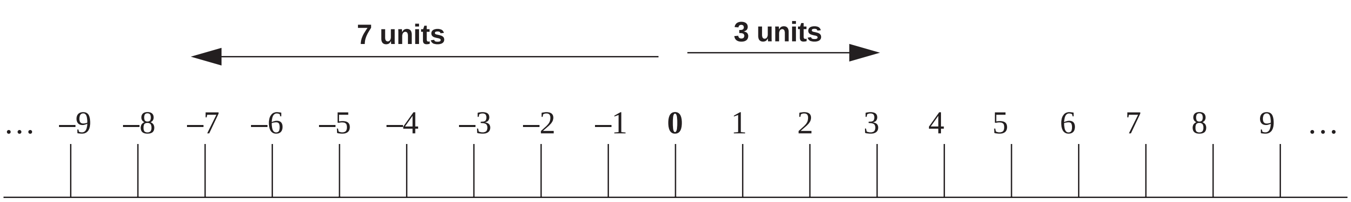
ABSOLUTE VALUE

Definition

Absolute value is the distance between any number on the number line and zero.

Examples

- The absolute value of –7 (“negative 7”) is 7, because –7 is 7 units from 0.
- The absolute value of 3 is 3, because 3 is located 3 units from 0.



ABSOLUTE VALUE (continued)

Absolute value is positive

Of course, a distance cannot be negative. Therefore, the absolute value of any signed number is always positive. Absolute value is *never negative*.

Note: The absolute value of zero is zero.

Symbol for absolute value

The symbol that indicates the absolute value of a number is a pair of vertical lines enclosing the number. The symbol looks like this: | |

Examples:
- "| 7 |" means the absolute value of 7 (which is 7).
- "| –7 |" means the absolute value of –7 (which is 7).

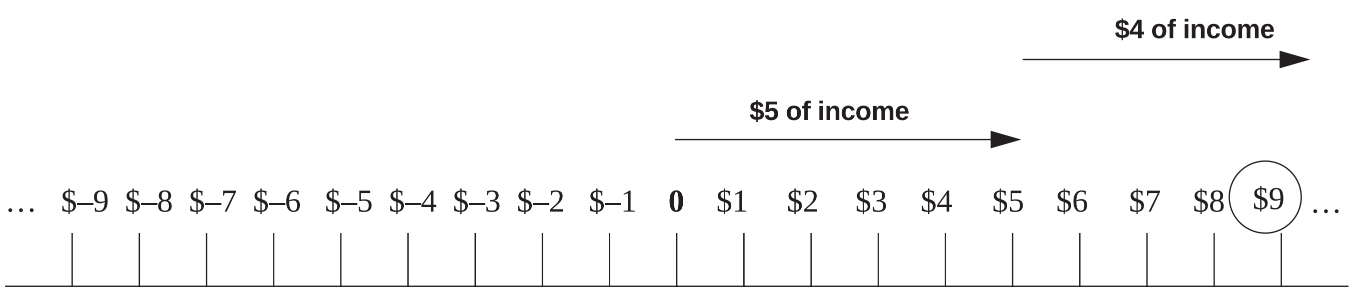
ADDITION OF SIGNED NUMBERS

Examples

The following four examples illustrate the addition of signed numbers by using income (which is positive) and expense (which is negative).

Example #1

- Suppose that you have a small business. You earn income from a sale of $5 (which is positive) and you earn income from another sale of $4 (which is also positive). What is the total effect on the wealth of the business?

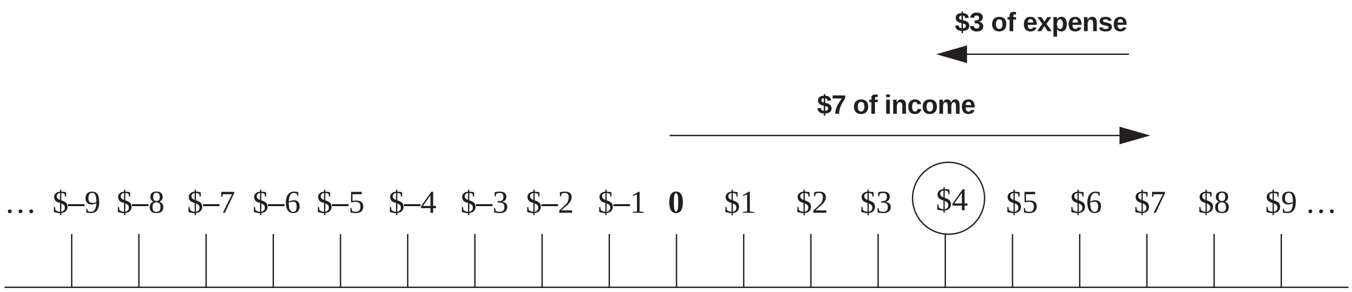


A positive $5 of income plus a positive $4 of income totals to a $9 increase in wealth (5 + 4 = 9).

ADDITION OF SIGNED NUMBERS (continued)

Example #2

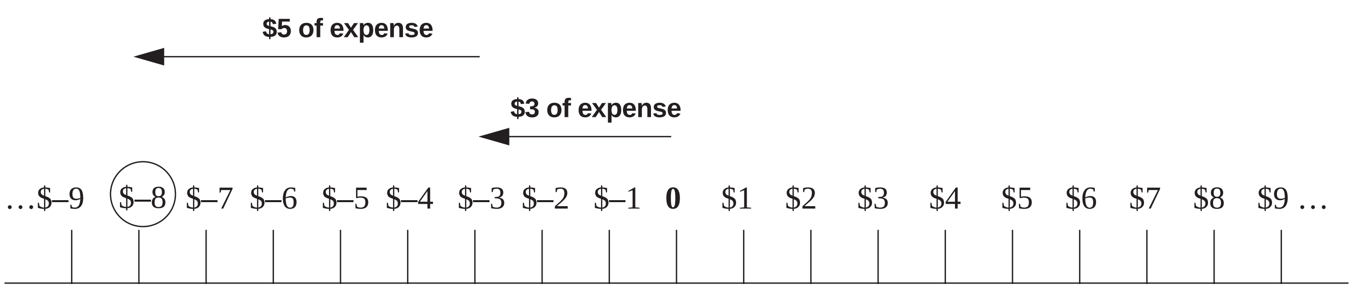
- Suppose you earn $7 of income from a sale but you also have an expense of $3. What total effect do these two amounts have on the wealth of the business?



$3 of expense (a negative) added to a positive $7 of income partially offsets the income. The total of these two items results in $4 of wealth (7 + (–3) = 4). Adding numbers with opposite signs shows how naturally opposite things will offset each other.

Example #3

- Suppose that the business has an expense of $3 and another expense of $5, but has not yet earned any income. What is the total effect on the business wealth of both expenses?

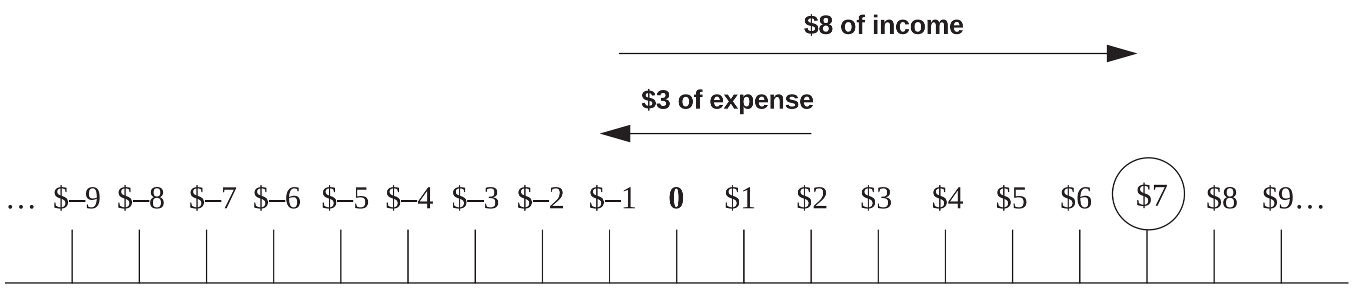


The expense of $3 plus another expense of $5 total to $–8. The wealth of the business was reduced by a total of $8 (–3 + (–5) = –8). Adding a negative to another negative results in a bigger negative total.

ADDITION OF SIGNED NUMBERS (continued)

Example #4

- Suppose that your business up to now has $2 of net profit. What is the total profit or loss if the business now incurs $3 of expenses followed by $8 of income?



In this case, our starting point is a positive $2, instead of 0. Beginning with the positive $2, a negative $3 is added. The result is a negative $1 (a $1 loss). However, when a positive $8 of income is added, the final result is a positive $7 profit (2 + (–3) + 8 = 7).

Rules for addition

The following table shows you the rules for adding signed numbers.

Rule	Examples
Same sign: To add two numbers with the same sign, add the absolute values, and then attach the common sign to the result.	• 7 + 4 = | 7 | + | 4 | = 11 attaching the common sign, the answer is 11. • –7 + (– 4) = | –7 | + | – 4 | = 11 attaching the common sign, the answer is –11.
Different signs: To add two numbers with different signs, subtract the smaller absolute value from the larger absolute value, and attach the sign of the number with the larger absolute value.	• 7 + (– 4) = | 7 | – | – 4 | = 3 attaching the sign of larger number, the answer is 3. • –7 + (4) = | –7 | – | 4 | = 3 attaching the sign of larger number, the answer is –3. *Note:* Notice how subtraction achieves the effect of combined opposites offsetting each other.
Any number + zero: The sum of any number and zero is the same number.	• 7 + 0 = 7 • –5 + 0 = –5
Any number + its opposite: The sum of any number and its opposite is zero.	• 7 + (–7) = 0 • –7 + 7 = 0

PRACTICE

SOLUTIONS FOR ADDING SIGNED NUMBERS BEGIN ON PAGE 106.

REINFORCEMENT PROBLEM: ADDING SIGNED NUMBERS

1. Calculate the totals of the expressions shown in the table:

Expression	Answer
a. 8 + 3	
b. 9 + (–3)	
c. –12 + (–10)	
d. –7 + 7	
e. –22 + (–30)	
f. –5 + (–9)	
g. 7 + 14	
h. 11 + 0	
i. 0 + 0	
j. –3 + 15	
k. 0 + (–5)	
l. 10 + (–20)	
m. –10 + (–20)	
n. 3 + (–3)	
o. –30 + 5	
p. –8 + (–3)	
q. –38 + (–9) + 84	
r. 21 + (–15) + (–38)	
s. –39 + (–12) + (–45)	

SOLUTIONS

PRACTICE QUESTIONS FOR ADDING SIGNED NUMBERS BEGIN ON PAGE 105.

REINFORCEMENT PROBLEM: ADDING SIGNED NUMBERS

1.

Expression	Answer
a. 8 + 3	11
b. 9 + (–3)	6
c. –12 + (–10)	–22
d. –7 + 7	0
e. –22 + (–30)	–52
f. –5 + (–9)	–14
g. 7 + 14	21
h. 11 + 0	11
i. 0 + 0	0
j. –3 + 15	12
k. 0 + (–5)	–5
l. 10 + (–20)	–10
m. –10 + (–20)	–30
n. 3 + (–3)	0
o. –30 + 5	–25
p. –8 + (–3)	–11
q. –38 + (–9) + 84	37
r. 21 + (–15) + (–38)	–32
s. –39 + (–12) + (–45)	96

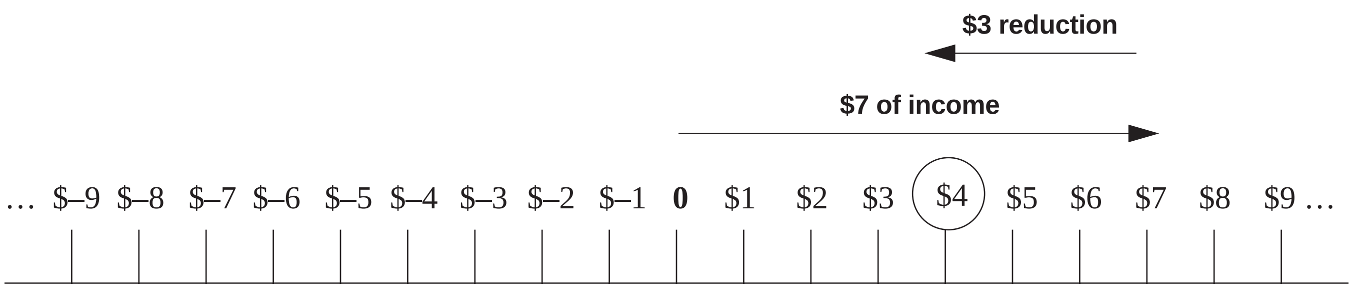
SUBTRACTION OF SIGNED NUMBERS

Examples

The following four examples illustrate the subtraction of signed numbers by using income (which is positive) and expense (which is negative).

Example #1

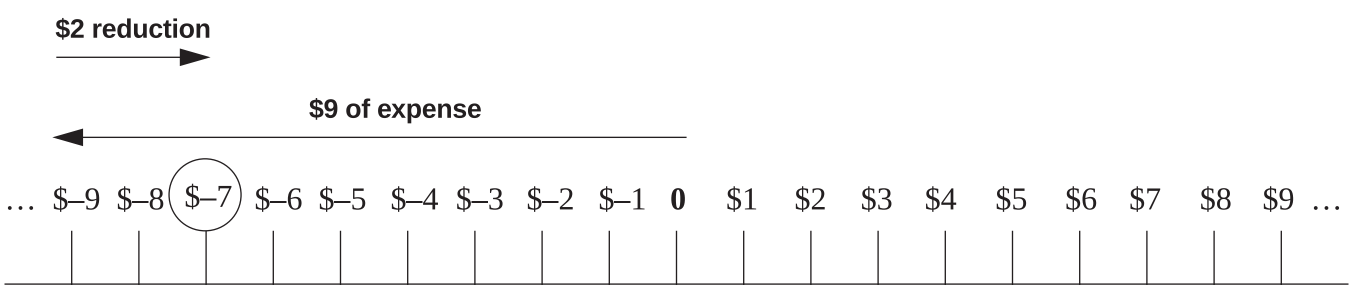
Suppose that you have a small business and that you earned $7 of income. Then an unhappy customer cancels a $3 sale, so now you have less income. How much income do you have now?



$7 of (positive) income minus the $3 decrease in the (positive) income leaves $4 of income, or 7 – 3 = 4.

Example #2

Suppose that your business has recorded $9 of expense (a negative item). However, now you discover that $2 of expense was recorded in error, so now you have less expense. How much expense do you have now?



$9 of a negative item (the expense) is reduced by $2, resulting in $7 of expense or, in other words, –9 – (–2) = –7.

SUBTRACTION OF SIGNED NUMBERS (continued)

Example #3

Mechanically, subtraction is used to calculate the effects of *adding more of a negative (opposite) thing*. (See “Rules for addition” on page 104.)

For example, suppose that your business has earned $7 of income. Next, the business pays a $3 expense. The business now has *more of a negative thing* (an expense) which must be combined with the income to calculate the net profit or loss. So, 7 income + 3 expense = 4 profit.

Notice that the final result of adding more of a negative (the expense) is exactly the same as subtracting a positive (the income) of the same amount.

Adding more of a negative ($3 of expense) has exactly the same result on profit as subtracting a positive ($3 of revenue):

Add more of a negative	Subtract a positive
7 + (–3) = 4	7 – 3 = 4

Rule for subtraction

Distinguishing between adding opposite things and subtracting similar things can be confusing. Instead, it is much easier to use the following *simple rule* whenever you see a minus sign between two numbers.

The following table shows you the rule for subtracting signed numbers with examples:

Rule	Example	Apply the rule …
Subtraction: To subtract two signed numbers, *add the opposite* of the number that is being subtracted. *Note:* Do *not* make any change to the number from which you are subtracting.	• 24 – 10 • 15 – (–20) • –15 – (–20) • –15 – 20 • 10 – 12	• 24 + (–10) = 14 • 15 + 20 = 35 • –15 + 20 = 5 • –15 + (–20) = –35 • 10 + (–12) = –2

SUBTRACTION OF SIGNED NUMBERS (continued)

Terminology: "subtract" and "from"

Example: "Subtract 15 from 12."
When you read or hear this kind of expression, it means that the number after the word "subtract" is being subtracted from the number after the word "from." In this case, it means: 12 – 15.

Other examples:
- "Subtract 22 from 100" means 100 – 22.
- "Subtract negative 9 from 15" means 15 – (–9).
- "From 33 subtract 12" means 33 – 12.

PRACTICE

SOLUTIONS FOR SUBTRACTING SIGNED NUMBERS BEGIN ON PAGE 111.

REINFORCEMENT PROBLEMS: SUBTRACTING SIGNED NUMBERS

1. Calculate the answer to each expression shown in the table below:

Expression	Answer	Expression	Answer
a. 12 – 3		i. 3 – (–1)	
b. 15 – 20		j. 30 – 20	
c. –9 – 9		k. 0 + (–5)	
d. –2 – (–15)		l. 10 – (–12)	
e. 0 –3		m. –5 – (–3)	
f. 15 –15		n. 5 – 5	
g. 15 – (–15)		o. 25 – 5	
h. –5 – 5		p. –8 – (–3)	

2. Calculate the answer to each of the following sentences:
   a. Subtract ten from thirty.
   b. Subtract twenty-five from three.
   c. Subtract a negative nine from five.
   d. Subtract a negative three from a negative eight.

3. **Addition and subtraction.** Calculate the answer to each expression in the table below:

Expression	Answer
a. 10 + 15 – 5	
b. –5 + (–7) – 3	
c. 5 – (–7) – (–5)	
d. –10 + (–3) – 8 + (–10)	
e. –4 + 3 – 2	
f. 30 – 15 + 8	
g. 15 + (–15)	
h. –50 – 10 + 12	
i. –5 + (–8) + 7 – 20	
j. 12 + (–5) – (–2)	
k. –8 – 2 + 4	
l. –5 – 7 –10	
m. –25 + 5 + 10	
n. 25 – (–5)	

SOLUTIONS

PRACTICE QUESTIONS FOR SUBTRACTING SIGNED NUMBERS BEGIN ON PAGE 110.

REINFORCEMENT PROBLEMS: SUBTRACTING SIGNED NUMBERS

1.

Expression	Answer
a. 12 – 3	9
b. 15 – 20	–5
c. –9 – 9	0
d. –2 – (–15)	13
e. 0 –3	–3
f. 15 –15	0
g. 15 – (–15)	30
h. –5 – 5	0
i. 3 – (–1)	4
j. 30 –20	10
k. 0 – (–5)	5
l. 10 – (–12)	22
m. –5 – (–3)	–2
n. 5 – 5	0
o. 25 –5	20
p. –8 – (–3)	–5

2. a. Subtract ten from thirty = 20
   b. Subtract twenty-five from three = –22
   c. Subtract a negative nine from five = 14
   d. Subtract a negative three from a negative eight = –5

3.

Expression	Answer
a. 10 + 15 – 5	20
b. –5 + (–7) – 3	–15
c. 5 – (–7) – (–5)	17
d. –10 + (–3) – 8 + (–10)	–31
e. –4 + 3 – 2	–3
f. 30 – 15 + 8	23
g. 15 + (–15)	0
h. –50 – 10 + 12	–48
i. –5 + (–8) + 7 – 20	–26
j. 12 + (–5) – (–2)	9
k. –8 – 2 + 4	–6
l. –5 – 7 –10	–22
m. –25 + 5 + 10	–10
n. 25 – (–5)	30

MULTIPLICATION OF SIGNED NUMBERS

Rules for multiplication

The table below shows you how to determine the sign of a product of two numbers:

Rule	Examples
Same signs: If the two numbers have the same sign, the product is positive.	• (3) (7) = 21 (two positives) • (–3) (–7) = 21 (two negatives)
Different signs: If the two numbers have different signs, the product is negative.	• (–3) (7) = –21 (negative and positive) • (3) (–7) = –21 (positive and negative)

Visual memory aid

(+) (+) = +
(–) (–) = +
(+) (–) = –
(–) (+) = –

Multiplying by 1 and by 0

The table below shows you some other important multiplication rules that involve multiplying by 1, by minus 1, and by 0.

Rule	Examples
Multiply by 1: The product of 1 and any number is that number.	• (1) (27) = 27 • (1) (22.459) = 22.459 • (1) (–7) = –7
Multiply by –1: The product of –1 and any number is that number, but with the opposite sign.	• (–1) (27) = –27 • (–1) (22.459) = –22.459 • (–1) (–7) = 7
Multiply by 0: The product of 0 and any number is 0.	• (0) (27) = 0 • (0) (22.459) = 0 • (0) (–7) = 0

DIVISION OF SIGNED NUMBERS

Rules for division

The table below shows you how to determine the sign of the quotient of two signed numbers.

Rule	Examples
Same signs: If the two numbers have the same sign, their quotient is positive.	• 20 / 5 = 4 • –20 / –5 = 4
Different signs: If the two numbers have different signs, their quotient is negative.	• 20 / –5 = – 4 • –20 / 5 = – 4

Visual memory aid

(+) ÷ (+) = +
(–) ÷ (–) = +
(+) ÷ (–) = –
(–) ÷ (+) = –

Division with 1 and 0 and the same number

The table below shows you some other important division rules that involve dividing by 1 and by 0.

Rule	Examples
Divide by 1: The quotient of any number divided by 1 is that number.	• (27) ÷ (1) = 27 • (22.459) ÷ (1) = 22.459 • (–7) ÷ (1) = –7
Divide by 0: The quotient of any number divided by zero cannot be determined, and is said to be “undefined.”	• (27) ÷ (0) = undefined • (22.459) ÷ (0) = undefined • (–7) ÷ (0) = undefined
Divide into 0: The quotient of zero divided by any number other than zero is zero.	• 0 ÷ 27 = 0 • (0) ÷ (–7) = 0
Divide by the same number: Any number (except zero) divided by itself is 1.	• 5 ÷ 5 = 1 • .000719 ÷ .000719 = 1

PRACTICE

REINFORCEMENT PROBLEMS: MULTIPLYING AND DIVIDING SIGNED NUMBERS

1. Calculate the product of each expression:

Expression	Answer
a. (7) (3)	
b. (–5) 4	
c. (–5) (–4)	
d. (–5) (–4) (–2)	
e. (8) (0)	
f. 8 (–8)	
g. (2) (–8) (–3)	
h. (8) (–1)	
i. –7 (–7)	
j. –7 (7)	
k. 5 (–3) (4)	
l. (8) • (4)	
m. (–8) • (–4)	
n. (–8) • (4)	
o. (–5) (–2) (–4)	
p. (–5) (–2) (–4) (–2)	

2. Calculate the quotient of each expression.

Expression	Answer
a. 8 ÷ 4	
b. –8 ÷ (–4)	
c. –8 ÷ (4)	
d. $^{14}/_{-2}$	
e. $^{-25}/_{5}$	
f. $^{-15}/_{-15}$	
g. $^{12}/_{-3}$	
h. $^{-12}/_{-3}$	
i. $^{7}/_{0}$	
j. $^{-8}/_{5}$	
k. 30 ÷ 5 ÷ 2	
l. (–30) ÷ (–5) ÷ (–2)	
m. 0 ÷ 5	
n. –18 ÷ (–9)	
o. –20 ÷ (–4)	
p. 20 ÷ (–4)	

SOLUTIONS

REINFORCEMENT PROBLEMS: MULTIPLYING AND DIVIDING SIGNED NUMBERS

1.

Expression	Answer
a. (7) (3)	21
b. (–5) 4	–20
c. (–5) (–4)	20
d. (–5) (–4) (–2)	–40
e. (8) (0)	0
f. 8 (–8)	–64
g. (2) (–8) (–3)	48
h. (8) (–1)	–8
i. –7 (–7)	49
j. –7 (7)	–49
k. 5 (–3) (4)	–60
l. (8) • (4)	32
m. (–8) • (–4)	32
n. (–8) • (4)	–32
o. (–5) (–2) (–4)	–40
p. (–5) (–2) (–4) (–2)	80

2.

Expression	Answer
a. 8 ÷ 4	2
b. –8 ÷ (–4)	2
c. –8 ÷ (4)	–2
d. $^{14}/_{-2}$	–7
e. $^{-25}/_{5}$	–5
f. $^{-15}/_{-15}$	1
g. $^{12}/_{-3}$	–4
h. $^{-12}/_{-3}$	4
i. $^{7}/_{0}$	undefined
j. $^{-8}/_{5}$	–1.6
k. 30 ÷ 5 ÷ 2	3
l. (–30) ÷ (–5) ÷ (–2)	–3
m. 0 ÷ 5	0
n. –18 ÷ (–9)	2
o. –20 ÷ (–4)	5
p. 20 ÷ (–4)	–5

MATHEMATICAL EXPRESSIONS

Definition

A **mathematical expression** is an arrangement of any numerals, letters, grouping symbols, and operational symbols which describes a value.

Symbols you are familiar with

Many of the symbols in a mathematical expression you already know:

- the numerals: 0, 1, 2, 3, 4, …
- common operational symbols such as add (+), subtract (–), multiply (•), and divide (÷)
- symbols indicating positive (+) and negative (–)

The equals symbol ("=")

The "=" symbol is very familiar, but technically it is not really part of a mathematical expression. Rather, the "=" shows the **equality** of two mathematical expressions.

Examples:

1) $4 + 8 = 4 \bullet 3$ shows that the expression of 4 plus 8 is equal to the expression of 4 times 3. Each expression has the value 12.

2) $\frac{21}{3} = 12 - 5$ shows the expression of 21 divided by 3 is equal to the expression of 12 minus 5. Each expression has the value 7.

Variable symbols

Letters used in mathematical expressions are called **variables**. Whenever some amount is not known, a letter (a variable) is written into the expression in place of the unknown amount. Variables are used frequently in algebra.

Examples of variables

$4 + x = 7$ says that: "4 plus some amount is equal to 7."
$4 + x$ is one mathematical expression and 7 is the other.

$\frac{y}{10} = 3 + x$ are expressions that show two unknown amounts. These two expressions say: "Some amount y divided by 10 is equal to 3 plus some other amount x."

MATHEMATICAL EXPRESSIONS (continued)

One of a variable

When a letter, such as x, is written by itself, it is understood that this represents 1 of x. This is the same as saying “1 times x.” For example, writing “x” means exactly the same thing as writing “$1x$.” For convenience, the numeral 1 is usually dropped.

Caution: Notice that “$1x$” is *NOT* the same as “$1 + x$,” which means 1 plus x.

Grouping symbols

How numbers are grouped makes a big difference in a calculation. The most common grouping symbol is the parenthesis “(   ).” Other grouping symbols are the “{   }” and “[   ]” brackets, and fraction bars “——” or “ / ”.

Examples of grouping symbol

$(4 + 3) \bullet 2$
The 4 and 3 are grouped together and add to 7. So, the expression results in 7 times 2, which is 14.

$4 + (3 \bullet 2)$
Notice what a difference it makes if the 3 and 2 were grouped together. In this case, the 3 is multiplied by 2 and results in 6. When 6 is added to 4 the answer is 10.

Note: Operations within the parentheses are always done first.

More examples of mathematical expressions

- 25 is a numerical value, and therefore is a mathematical expression.
- $(x + 3)$ expresses the total of some amount plus 3.
- $25x - (x + 3)$ is a combination of symbols in an mathematical expression.

This expression says “25 times an unknown amount x minus the sum of the unknown amount plus 3.”

Basic operation symbols

When studying arithmetic, you learned the four basic operations of addition, subtraction, multiplication, and division. Here are examples of expressions using each operational symbol with the numeral 3 and the variable x.

MATHEMATICAL EXPRESSIONS (continued)

Caution!

Notice that these two are different!

Operation	Mathematical Expression	English Descriptions
Addition	$3 + x$, or $(3 + x)$	• 3 plus x • x more than 3 • the total of 3 plus x
Subtraction	$x - 3$, or $(x - 3)$	• x minus 3 • 3 less than x • 3 subtracted from x
Multiplication	$3x$, or $3 \bullet x$, or $(3)(x)$, or $3(x)$, or $(3)x$	• 3 times x • 3 multiplied by x • x multiplied by 3 • 3 of x
Division	$\frac{x}{3}$ or $x/3$ or $x \div 3$	• x divided by 3 • 3 divided into x

EXPONENTS

Exponents

An **exponent** is a small numeral placed slightly behind and above another numeral, to indicate that the preceding numeral must be multiplied by itself. For example, in the expression 3^2 the two indicates that 3 must be multiplied by itself twice. This is read as "three to the second power," or "three squared." The calculation is: (3) (3) = 9.

More examples

- 3^3 is read "three to the third power," which is (3) (3) (3) = 27.
- 3^4 is read "three to the fourth power," which is (3) (3) (3) (3) = 81.
- x^4 is read "x to the fourth power," which is $(x)(x)(x)(x)$.
- $3x^4$ is read "x to the fourth power, multiplied by three."

Caution

An exponent only applies to the value that **immediately precedes it**.

Examples

- $3 + 2^3 = 11$
- $(3 + 2)^3 = 125$
- $-3^2 = -9$ (the negative of 3 times 3)
- $(-3)^2 = 9$ (-3 times -3)
- $3x^2 = (3)(x)(x)$

PRACTICE

REINFORCEMENT PROBLEM: CALCULATING WITH EXPONENTS

1. Calculate the value of the following expressions:

Expression	Answer
a. 5^2	
b. 5^3	
c. -5^2	
d. -5^3	
e. $-(-5)^3$	
f. $(4-7)^2$	
g. $(-2)^3 + 5^2$	
h. $(-6+8)^3 + (7+1)^2$	
i. -2^4	
j. $4^2 - 2^2$	
k. $-(-10)^2$	
l. $(x+y)^2$ when $x = 2$ and $y = 3$	
m. $x + y^2$ when $x = 2$ and $y = 3$	
n. $-y^2$ when $y = 2$	
o. $2^4/2^2$	
p. $-(-4)^2 - (-5)^2$	

SOLUTIONS

REINFORCEMENT PROBLEM: CALCULATING WITH EXPONENTS

1.

Expression	Answer
a. 5^2	25
b. 5^3	125
c. -5^2	–25
d. -5^3	–125
e. $-(-5)^3$	125
f. $(4-7)^2$	9
g. $(-2)^3 + 5^2$	17
h. $(-6+8)^3 + (7+1)^2$	72
i. -2^4	–16
j. $4^2 - 2^2$	12
k. $-(-10)^2$	–100
l. $(x+y)^2$ when $x = 2$ and $y = 3$	25
m. $x + y^2$ when $x = 2$ and $y = 3$	11
n. $-y^2$ when $y = 2$	– 4
o. $2^4/2^2$	4
p. $-(-4)^2 - (-5)^2$	– 41

EVALUATING EXPRESSIONS

Introduction

Now that you have reviewed mathematical expressions and the key symbols in those expressions, you are ready to begin calculating the values of the expressions. This is an extremely important skill. You should practice it until you feel confident every time you do it.

Definition: "evaluate"

In mathematics, to **evaluate** an expression means to determine its value.

Order of operations

There are two elements involved in successfully evaluating an expression:

- a clear understanding of the mathematical operations
- knowing the *correct order* for performing these operations

The table below shows you the order of the steps that are performed to evaluate a mathematical expression. The operation to be evaluated is highlighted with a shaded box.

Step	Action	Examples
1	**Parenthesis ( ):** Evaluate the operations *within* parentheses or other grouping symbols. If there are two or more operations within a grouping symbol, do them in the order of Steps 2, 3, and 4 below.	*Evaluate:* $(9 \bullet 2 + 20) \bullet 3 - 5 \div 2^3$ $(\boxed{9 \bullet 2} + \boxed{20}) \bullet 3 - 5 \div 2^3$ *Result:* $(38) \bullet 3 - 5 \div 2^3$
2	**Exponents:** Evaluate all the exponents.	*Evaluate:* $(38) \bullet 3 - 5 \div 2^3$ $38 \bullet 3 - 5 \div \boxed{2^3}$ *Result:* $38 \bullet 3 - 5 \div 8$
3	**Multiply and Divide:** Evaluate all the multiplication and division operations as they occur from *left to right*.	*Evaluate:* $38 \bullet 3 - 5 \div 8$ $\boxed{38 \bullet 3} - \boxed{5 \div 8}$ *Result:* $114 - .625$
4	**Add and Subtract:** Evaluate all the addition and subtraction operations as they occur from *left to right*.	*Evaluate:* $114 - .625$ $\boxed{114 - .625}$ *Result:* 113.375

EVALUATING EXPRESSIONS (continued)

Follow the steps exactly

Knowing which operation to perform is an important skill that will always help you. The trick is to **follow the procedure exactly**. No matter how complicated the operations seem to be, if you slowly and carefully follow the procedure you will get the right answer.

Division bar is also a grouping symbol

If you see a division bar symbol, such as $\frac{20}{5 \bullet 2}$ or 20 / 5 • 2, treat the bar as a grouping symbol. This means that any expressions above or below the division bar are evaluated first as part of Step 1, just as if they were within parentheses, before any division is done. In this example, the 5 and 2 are evaluated first, to obtain 10. So, the final operation is 20 divided by 10.

Memory aid

A good memory aid for remembering the steps is to notice that the first letters of the operation steps spell the word "PEMDAS" (**P**arenthesis, **E**xponent, **M**ultiply, **D**ivide, **A**dd, and **S**ubtract). There are lots of cute expressions to help you remember the "PEMDAS" letters. One of them is "Popcorn Every Minute Doesn't Always Satisfy." Another real original is "Please Exhume Mr. Dracula At Sunrise."

Evaluating operations with variables

The use of letters (that is, variables) has no effect on the order of operations. The operations are performed in exactly the same steps. The only difference is that you must be given some value for each of the variables in order to determine the numerical value of the entire expression.

Example with variables

Evaluate: $4 \bullet y^2 - 10 + (8 - x) / 2$, when $y = .5$ and $x = 4$.
Therefore, substitute the value .5 for y and the value 4 for x.

The result is $4 \bullet .5^2 - 10 + (8 - 4) / 2$

Step	Action	Examples
1	**Parenthesis:** The only operation within the ( ) is to subtract 4 from 8.	$4 \bullet .5^2 - 10 + (8 - 4) / 2$ becomes $4 \bullet .5^2 - 10 + 4 / 2$
2	**Exponents:** Multiply .5 times .5, which results in .25.	$4 \bullet .5^2 - 10 + 4 / 2$ becomes $4 \bullet .25 - 10 + 4 / 2$
3	**Multiply and Divide:** Moving from left to right, multiply 4 times .25, and then divide 4 by 2.	$4 \bullet .25 - 10 + 4 / 2$ becomes $1 - 10 + 2$
4	**Add and Subtract:** Moving from left to right, subtract 10 from 1, which is –9, and then add 2 to –9.	$1 - 10 + 2$ becomes -7

PRACTICE

SOLUTIONS FOR EVALUATING EXPRESSIONS BEGIN ON PAGE 122.

REINFORCEMENT PROBLEMS: EVALUATING EXPRESSIONS

1. Evaluate each of the expressions and write your answer next to the expression.

Expression	Answer	Expression	Answer
a. $10 - 7 + 2$		l. $\left[\frac{10}{2} \bullet \frac{4}{2}\right]^2$	
b. $10 - 7 \bullet 2$		m. $\left[\frac{1}{2} \bullet \frac{4}{10}\right] + 1.5^2$	
c. $(10 - 7) \bullet 2$		n. $\left(-.8^2 + \frac{3}{10}\right) \bullet \left[\left(\frac{10}{5}\right)^2\right]^2$	
d. $(10 - 7)^2 \bullet 2$		o. $81 - (5 - 8)^2 \big/ 3 + 3^2 \div 3$	
e. $(5 - 3) + (7 + 3) / 2$		p. $8 \div 4 \bullet 8 \bullet 3^2$	
f. $(5 - 3^2)^2 + (7 + 3) / 2$		q. $[8 - (-3 - 2)]^2$	
g. $9 \bullet 4^2 + 5 \bullet 8$		r. $4^2 + 1 \big/ -3^2 + (-3)^2$	
h. $2 \bullet 3^3 - 10 \bullet 3$		s. $\frac{2 + 3 \bullet 2^3}{-8} - (3 \bullet 3)$	
i. $36 + (4^2 - 2) / 3 + 2^2$		t. $3 - (10 + 2)^2 \big/ 3^2 - 12$	
j. $(2^3 \bullet 5 - 1)(2^2 + 1)$		u. $120 \div (-4) / 2$	
k. $5 + (3^3 - 17)^3 - 5$		v. $5 - 35 \bullet 2 / 2 + 3 \bullet 8$	

2. Evaluate each of the expressions and write your answer next to the expression.

Expression	Answer	Expression	Answer
a. $4x + 3$ when $x = 2$		i. $4r^2 (r - 1)$ when $r = -3$	
b. $4(x + 3)$ when $x = 2$		j. $3(x + y)$ when $x = 4, y = -8$	
c. $4(x + 3)^2$ when $x = 2$		k. $3(x - y) \big/ (y - 1)$ when $x = 9, y = 3$	
d. $-4x^2 (x + 3)^2$ when $x = 2$		l. $x^3 \big/ x^2$ when $x = 2$	
e. $-x^2 - 5x + 5$ when $x = 3$		m. $(x - y)^2$ when $x = 5, y = 2$	
f. $8y - 3$ when $y = 4$		n. x^y when $x = 2, y = 3$	
g. $y^2 - 5$ when $y = 2$		o. $p = r \bullet b$ when $r = .2, b = 100$	
h. $2a^2 - 5a + 4$ when $a = -2$		p. $a = l + e$ when $l = 2{,}000, e = 750$	

SOLUTIONS

PRACTICE QUESTIONS FOR EVALUATING EXPRESSIONS BEGIN ON PAGE 121.

REINFORCEMENT PROBLEMS: EVALUATING EXPRESSIONS

1.

Expression	Answer	Expression	Answer
a. $10 - 7 + 2$	5	l. $\left[\frac{10}{2} \bullet \frac{4}{2}\right]^2$	100
b. $10 - 7 \bullet 2$	–4	m. $\left[\frac{1}{2} \bullet \frac{4}{10}\right] + 1.5^2$	2.45
c. $(10 - 7) \bullet 2$	6	n. $\left(-.8^2 + \frac{3}{10}\right) \bullet \left[\left(\frac{10}{5}\right)^2\right]^2$	–5.44
d. $(10 - 7)^2 \bullet 2$	18	o. $81 - (5 - 8)^2 \big/ 3 + 3^2 \div 3$	12
e. $(5 - 3) + (7 + 3) / 2$	7	p. $8 \div 4 \bullet 8 \bullet 3^2$	144
f. $(5 - 3^2)^2 + (7 + 3) / 2$	21	q. $[8 - (-3 - 2)]^2$	169
g. $9 \bullet 4^2 + 5 \bullet 8$	184	r. $4^2 + 1 \big/ -3^2 + (-3)^2$	undefined
h. $2 \bullet 3^3 - 10 \bullet 3$	24	s. $\frac{2 + 3 \bullet 2^3}{-8} - (3 \bullet 3)$	–12.25
i. $36 + (4^2 - 2) / 3 + 2^2$	38	t. $3 - (10 + 2)^2 \big/ 3^2 - 12$	47
j. $(2^3 \bullet 5 - 1)(2^2 + 1)$	195	u. $120 \div (-4) / 2$	–15
k. $5 + (3^3 - 17)^3 - 5$	1,000	v. $5 - 35 \bullet 2 / 2 + 3 \bullet 8$	–2.5

2.

Expression	Answer	Expression	Answer
a. $4x + 3$ when $x = 2$	11	i. $4r^2 (r - 1)$ when $r = -3$	–144
b. $4 (x + 3)$ when $x = 2$	20	j. $3 (x + y)$ when $x = 4, y = -8$	–12
c. $4 (x + 3)^2$ when $x = 2$	100	k. $3 (x - y) \big/ (y - 1)$ when $x = 9, y = 3$	9
d. $-4x^2 (x + 3)^2$ when $x = 2$	–400	l. $x^3 \big/ x^2$ when $x = 2$	2
e. $-x^2 - 5x + 5$ when $x = 3$	–19	m. $(x - y)^2$ when $x = 5, y = 2$	9
f. $8y - 3$ when $y = 4$	29	n. x^y when $x = 2, y = 3$	8
g. $y^2 - 5$ when $y = 2$	–1	o. $p = r \bullet b$ when $r = .2, b = 100$	$p = 20$
h. $2a^2 - 5a + 4$ when $a = -2$	22	p. $a = l + e$ when $l = 2{,}000, e = 750$	$a = 2{,}750$

Introduction to Algebra and Equations

OVERVIEW

Introduction

After you have mastered arithmetic, algebra is the most important kind of mathematics that you will learn. This is because algebra has so many practical uses. There are many activities and conditions in the world that can be described and then quantified by using algebra.

Algebra is the technique of writing a statement that uses mathematical expressions to numerically describe a relationship, an activity, or a condition. By doing this, algebra can provide exact answers to questions about many activities or conditions.

Examples of applications

Here are just a few examples of the almost unlimited activities and conditions that algebra can describe and calculate answers for:

- the balance of an account
- time needed to travel a specific distance
- what your loan payments will be
- the cost of insurance
- the amount of sod needed for a new lawn
- dosage formulas for medicine
- the amount of sales needed to earn a specified profit
- formula for the theory of relativity ($e = mc^2$)
- a balance sheet for a business

and the list could go on and on …

OVERVIEW (continued)

Algebra compared to arithmetic

When someone studies algebra for the first time, that person often wants to know, "What is the difference between algebra and arithmetic?"

- In the operations for arithmetic, all the values that are used in the calculations are known and given to you. Each time you perform an operation or calculation in arithmetic, you know all the numerical values that you will use.
- Algebra uses the exact same kinds of calculations for the same exact situations that you learned in arithmetic. This is good news! What makes algebra different than arithmetic is that algebra generalizes the patterns or types of calculations without having to use specific numbers.

Algebra can be easily identified because in algebra, letters are often substituted for specific numbers, and an "=" sign is used. However, all of this is nothing more than a generalization of what you already learned in arithmetic.

As you will see, generalizing can be very, very powerful and very, very useful.

Variables

In algebra, missing or unknown values are represented by letters. These letters are called **variables**. Commonly used letters for variables are *x*, *y*, and *z*.

Examples

The table below shows some simple comparisons between arithmetic and algebra.

Arithmetic	Algebra	Difference
"5 + 3 is some amount"	$a + b = x$	Algebra generalizes the addition calculation. Any numbers can be substituted for *a* and *b* to get a correct answer, *x*.
"4 × 7 is some amount"	$a \bullet b = x$	Algebra generalizes the multiplication calculation. Any numbers can be substituted for *a* and *b* to get a correct answer, *x*.
"3 + 7 and 7 + 3 both equal 10"	$a + b = b + a$	Algebra generalizes the addition relationship for any numbers.

EQUATIONS

Definition

An **equation** is a statement which shows two mathematical expressions that are equal to each other. The symbol "=" is used to show the equality.

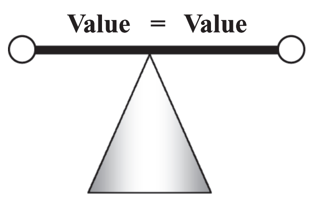
Examples

$10 + 2 = \frac{288}{24}$ is an equation.

$x + 5 = 3$ is an equation.

$p = b + r - 20$ is an equation.

The equation must always stay in balance



The most important thing about an equation is this: an equation must *always* remain in balance, no matter what. Think of an equation as a balance beam like you see below, with a value on each side of the beam. If some value is added to or removed from one side, the same thing must be done to the other side, or the beam will not balance.

TERMS OF AN EQUATION

Terms

When a mathematical expression in an equation has various parts, the parts of the equation that are *added or subtracted* are called **terms**.

Examples

The following table shows examples of various terms:

Mathematical Expression	Terms
$8 + 3 - x$	$8, 3, x$
$-4z + 8y$	$-4z, 8y$
$40 - (x + y) - 12x$	$40, (x + y), 12x$
$.7x + .9x - 3.5x$	$.7x, .9x, 3.5x$
$3(x + 8) - 5 + x^2$	$3(x + 8), 5, x^2$

TERMS OF AN EQUATION (continued)

Numerical coefficients

When a term is made up of both a variable and a numeral, the numeral part of the term is called a **numerical coefficient**. If a variable appears by itself, its numerical coefficient is 1, although by custom the 1 is not usually written.

Examples

The following table shows examples of terms and numerical coefficients:

Term	Numerical Coefficient	Which Means …
$5x$	5	Five times x
x	1	1 times x
x^2	1	1 times x^2
$-3x$	-3	-3 times x
$-x^2$	-1	-1 times x^2
$3(x + 8)$	3	3 times the quantity $(x + 8)$

Like terms

Like terms are those terms that have the same variables and same exponents. The coefficients may be different.

Examples

The following table shows examples of like terms and unlike terms:

Mathematical Expression	Like Terms	Unlike Terms
$3x + 4y + 7x$	$3x$, $7x$	$4y$
$8y + (-7y) + 5$	$8y$, $-7y$	5
$5x^2 + 3y^2$	none	$5x^2$, $3y^2$
$5x^2 + 3x^2$	$5x^2$, $3x^2$	none
$3y - 7 + 8x + 10$	7, 10	$3y$, $8x$
$3x + 2 - 5x + 8$	$3x$ and $5x$, 2 and 8	none

Note: Numerals by themselves are like terms and may always be combined.

SOLVING EQUATIONS—OVERVIEW

Definition

"To solve an equation" means to find the value of a variable which makes the equation a true statement.

How to know if you have a correct solution

The way you can always know if you have a correct solution to an equation is to do the following:

- Replace the variable with the solution value.
- Evaluate the equation to determine the numerical value on each side.
- If the two sides are equal—if the equation stays in balance—the solution is correct.

Examples

The following table shows examples of how to check equations. (We will use the "=?" notation while we are checking to see if the solution is correct.)

Equation	Proposed Solution	Check By Replacing Variable
$12 - x = 9$	3	$12 - \mathbf{3} =? 9$ $9 = 9$ (true) Yes, 3 is a correct solution.
$100 + x - 250 = 720$	870	$100 + \mathbf{870} - 250 =? 720$ $720 = 720$ (true) Yes, 870 is a correct solution.
$4x + 42 = -8x$	–3.5	$4(\mathbf{-3.5}) + 42 =? -8(\mathbf{-3.5})$ $-14 + 42 =? 28$ $28 = 28$ (true) Yes, –3.5 is a correct solution.
$150 - 3x + 25 = 275$	40	$150 - 3(\mathbf{40}) + 25 =? 275$ $150 - 120 + 25 =? 275$ $55 = 275$ (false) No, 40 is not a correct solution.

PRACTICE

REINFORCEMENT PROBLEM: CHECKING THE SOLUTION

1. Check the proposed solutions to each of the equations below. Write "yes" if the proposed solution makes the equation a true statement, or write "no" if it does not make the equation a true statement.

Equation	Proposed Solution	Solution Makes Equation a True Statement?
a. $2x + 3 = 11$	$x = 4$	
b. $6x^2 - 9 = 45$	$x = -3$	
c. $\frac{30}{(x-1)+2} = -3$	$x = -5$	
d. $(x-1)(x+2) = 13.75$	$x = 3.5$	
e. $\frac{x^5}{x^2} = 8$	$x = 2$	
f. $-x - 3x \bullet 10 = 10$	$x = -.5$	
g. $\frac{x^2+5}{x-5+3} = 39.2$	$x = 30$	
h. $x^2 + y - 12 = 15$	$x = 5, y = 3$	
i. $\frac{8(y-5)+x^2}{x-2} = -20$	$x = 4, y = -2$	

SOLUTIONS

REINFORCEMENT PROBLEMS: CHECKING THE SOLUTION

1.

Equation	Proposed Solution	Solution Makes Equation a True Statement?	
a. $2x + 3 = 11$	$x = 4$	yes	$8 + 3 = 11$ (true)
b. $6x^2 - 9 = 45$	$x = -3$	yes	$54 - 9 = 45$ (true)
c. $\frac{30}{(x-1)+2} = -3$	$x = -5$	no	$30 / -4 = -3$ (false)
d. $(x-1)(x+2) = 13.75$	$x = 3.5$	yes	$(2.5)(5.5) = 13.75$ (true)
e. $\frac{x^5}{x^2} = 8$	$x = 2$	yes	$32 / 4 = 8$ (true)
f. $-x - 3x \bullet 10 = 10$	$x = -.5$	no	$.5 + 15 = 10$ (false)
g. $\frac{x^2+5}{x-5+3} = 39.2$	$x = 30$	no	$905 / 28 = 39.2$ (false)
h. $x^2 + y - 12 = 15$	$x = 5, y = 3$	no	$16 = 15$ (false)
i. $\frac{8(y-5)+x^2}{x-2} = -20$	$x = 4, y = -2$	yes	$-40 / 2 = -20$ (true)

SOLUTION PROCEDURES

Introduction to solution procedure

Now that you know how to verify whether or not a solution to an equation is correct, it is time to begin learning *how to* solve an equation, to get the solution.

Solving equations is an extensive subject that will be continued in the next book of this accounting tutorial series. However, the following topics will show you how to deal with those equations that can be solved by simplifying and by adding and subtracting terms.

Overview of procedure steps

The goal is to have the variable isolated on one side of the equation. To accomplish this, we do the following, while always making sure that the equation stays in balance:

Step 1: Simplify the terms of the equation.
Step 2: Add or subtract the terms.

SIMPLIFYING TERMS

Overview

The terms of an equation can be simplified by doing the following:

- combining like terms
- removing parenthesis grouping whenever possible

Combining like terms

Combining like terms means to add or subtract the coefficients of whatever like terms appear on each side of an equation. The variable attaches to the result.

Example #1

Simplify $5x + 7x = 300$.

- $5x$ and $7x$ are like terms that have the same variable x.
- Adding the coefficients, we obtain 12.

Final result: $12x = 300$, which is a simpler equation.

SIMPLIFYING TERMS (continued)

Example #2

Simplify $x + x^2 - 20x = 5x - 12x$.

- Left side: x and $-20x$ are like terms that have the same variable x.
- Subtracting the coefficients, we obtain –19.
- Result on left side: $x^2 - 19x$.

- Right side: $5x$ and $-12x$ are like terms that have the same variable x.
- Subtracting the coefficients, we obtain –7.
- Result on right side: $-7x$.

Final result: $x^2 - 19x = -7x$, which is a simpler equation.

Remove parenthesis

If the amount within a parenthesis is a term that is being added to or subtracted from other terms, then it is often useful to know how to remove the parenthesis. This is because after the parenthesis has been removed, there will frequently be like terms that can then be combined, or numbers that can be evaluated.

Procedure

The following table shows you the procedure for removing a parenthesis, or similar grouping symbols such as [ ] or { }, when the parenthesis term is being added to or subtracted from other expressions.

If …	Then …	Examples
No sign or a plus sign directly precedes the parenthesis,	The parenthesis may simply be removed. (The expression inside the parenthesis remains unaffected.)	• $(4x - 2) - 3x = 14$ is changed to: $4x - 2 - 3x = 14$ • $12 + (-2y + 8) = (7 - 3y)$ is changed to: $12 - 2y + 8 = 7 - 3y$
The parenthesis is directly **preceded by a minus sign**,	Reverse all the signs of the terms within the parenthesis when the parenthesis is removed.	• $3 - (4x - 2) - 3x = 14$ is changed to: $3 - 4x + 2 - 3x = 14$ • $12 - (-2y + 8) = -(7 + 3y)$ is changed to: $12 + 2y - 8 = -7 - 3y$

SIMPLIFYING TERMS (continued)

Note: In the third example, notice how the positive $4x$ within the parenthesis changed to a negative $4x$, and the –2 within the parenthesis changed to a +2. In the fourth example, notice how all the signs of the terms within the parenthesis also have been reversed when the parenthesis was removed. Also notice how the new sign of the first term in the parenthesis replaces the sign directly in front of the parenthesis when the parenthesis is removed.

Caution

Do *not* try to remove the parenthesis if the parenthesis term is being multiplied or divided by another term. A different procedure is needed, which is presented in the second book in this series.

Examples:
- $3\ (x + 2) = 10$
- $(x + 4) / 5 = 12$

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PRACTICE

REINFORCEMENT PROBLEMS: SIMPLIFYING EXPRESSIONS

1. Remove parentheses where possible.

Equation	Rewritten Equation
a. $(2x + 10) - 3x = 230$	
b. $(4x - 3) + 10x = 50$	
c. $4x - (3 + 10x) = 50$	
d. $4\,(x - 3) + 10x = 50$	
e. $(2x + 10x) = x^2 - (3x - 8)$	
f. $(7x + 3) / 5x = 120 + x$	
g. $-(2x - 8) - (5y + 5) = 80$	

2. On a separate piece of paper, simplify the following equations by removing parentheses and combining like terms where possible.

Equation
a. $5 - x + 2x + 3 = 14$
b. $(5y - 3) - (2y + 10) = 20$
c. $50 = 5a - 6 + (10a - 12)$
d. $5\,(x - 3) + (5x + 2) = -30$
e. $-(x + 12 + 5x) + (x - 3) = 100$
f. $(x - 10) - (-x + 5) = 8$
g. $(x + 3)^2 - (x - 3) = 5$
h. $(a + b) - (5 - b) = -25$
i. $(x + 3) / 5 + 5 - 2 = 100$
j. $-75 = y^2 - 5y + 8 - (-4y - 10)$
k. $6 - 3x - 12 + 2x = 15$

SOLUTIONS

REINFORCEMENT PROBLEMS: SIMPLIFYING EXPRESSIONS

1.

Equation	Rewritten Equation
a. $(2x + 10) - 3x = 230$	$2x + 10 - 3x = 230$
b. $(4x - 3) + 10x = 50$	$4x - 3 + 10x = 50$
c. $4x - (3 + 10x) = 50$	$4x - 3 - 10x = 50$
d. $4\,(x - 3) + 10x = 50$	cannot remove parenthesis unless another procedure is used
e. $(2x + 10x) = x^2 - (3x - 8)$	$-2x + 10x = x^2 - 3x + 8$
f. $(7x + 3) / 5x = 120 + x$	cannot remove parenthesis unless another procedure is used
g. $-(2x - 8) - (5y + 5) = 80$	$2x + 8 - 5y - 5 = 80$

2.

Equation	Simplified Equation
a. $5 - x + 2x + 3 = 14$	$x + 8 = 14$
b. $(5y - 3) - (2y + 10) = 20$	$3y - 13 = 20$
c. $50 = 5a - 6 + (10a - 12)$	$50 = 15a - 18$
d. $5\,(x - 3) + (5x + 2) = -30$	$5\,(x - 3) + 5x + 2 = -30$
e. $-(x + 12 + 5x) + (x - 3) = 100$	$-5x - 15 = 100$
f. $(x - 10) - (-x + 5) = 8$	$2x - 15 = 8$
g. $(x + 3)^2 - (x - 3) = 5$	$(x + 3)^2 - x + 3 = 5$
h. $(a + b) - (5 - b) = -25$	$a + 2b - 5 = -25$
i. $(x + 3) / 5 + 5 - 2 = 100$	$(x + 3) / 5 + 3 = 100$
j. $-75 = y^2 - 5y + 8 - (-4y - 10)$	$-75 = y^2 - y + 18$
k. $6 - 3x - 12 + 2x = 15$	$-x - 6 = 15$

ADDING AND SUBTRACTING TERMS

Overview

Remember that the goal in solving an equation is to **isolate the variable** on one side of the equation. Up to now we have worked toward this objective by practicing procedures for simplifying an equation.

Now we will practice the next step: adding and subtracting terms. A very important way to isolate the variable and still maintain the equality of the equation is by adding or subtracting the same value to each side of the equation.

Procedure

The following table shows the procedure for adding and subtracting terms. The operation to be evaluated is highlighted with a shaded box.

> *Note:* This procedure applies to isolating a single variable that appears on one side of an equation.

<table>
<tr><th>Step</th><th colspan="2">Procedure</th><th>Example</th></tr>
<tr><td>1</td><td colspan="2">Locate the variable and identify which side of the equation it is on.</td><td>$x + 3 = 28$

The variable is x and it is on the left side.</td></tr>
<tr><td rowspan="4">2</td><td colspan="2">Remove the numbers on the same side of the equation as the variable.</td><td rowspan="2"></td></tr>
<tr><th>If …</th><th>Then …</th></tr>
<tr><td>the number is positive or added,</td><td>subtract it from both sides</td><td>$x + 3 = 28$
$x + 3 \mathbf{- 3} = 28 \mathbf{- 3}$
$x + 0 = 25$
$x = 25$</td></tr>
<tr><td>the number is negative or subtracted,</td><td>add it to both sides</td><td>$x - 10 = 30$
$x - 10 \mathbf{+ 10} = 30 \mathbf{+ 10}$
$x + 0 = 40$
$x = 40$</td></tr>
<tr><td>3</td><td colspan="2">Check the solution by replacing the variable with the solution value.</td><td>$x + 3 = 28$
$\mathbf{25} + 3 = 28$ (true)

$x - 10 = 30$
$\mathbf{40} - 10 = 30$ (true)</td></tr>
</table>

ADDING AND SUBTRACTING TERMS (continued)

More examples

- **The equation that describes the calculation of net income is:**

$r - e = i$, where r means revenue, e means expenses, and i means net income.

If the net income of our company was $1,000 and the expenses were $500, what were the revenues?

Solution:

$r - 500 = 1{,}000$	(values entered in equation)
$r - 500 + 500 = 1{,}000 + 500$	(removing the –500 on the side of the variable)
$r = 1{,}500$	(revenue)

Check: $1,500 – $500 = $1,000
$1,000 = $1,000 (true)

- **The equation that shows the ending balance of an account is:**

$b + i - d = e$, where the b means beginning balance, i means account increases, d means account decreases and e means ending balance.

Suppose that the beginning balance of the cash account was $10,000 and the company made cash payments of $47,000 during the month. The month-end cash balance was $2,500. What were the cash receipts during the month?

Solution:

$10{,}000 + i - 47{,}000 = 2{,}500$	(values entered in equation)
$10{,}000 - 10{,}000 + i - 47{,}000 = 2{,}500 - 10{,}000$	(isolating the variable)
$i - 47{,}000 = -7{,}500$	
$i - 47{,}000 + 47{,}000 = -7{,}500 + 47{,}000$	(isolating the variable)
$i = 39{,}500$	(the cash receipts)

Check: $10,000 + $39,500 – $47,000 = $2,500
$2,500 = $2,500 (true)

ADDING AND SUBTRACTING TERMS (continued)

When the isolated variable is negative

Suppose that you have this equation: $10 - x = 25$

Solution:
$10 - 10 - x = 25 - 10$
$0 - x = 15$
$-x = 15$

It may appear that the solution to this equation is $-x = 15$; however, that is not the case, because we are solving for x (positive x), not $-x$. From our discussion of signed numbers, we know that $-x$ means the negative of x. Therefore, the solution (the positive of x) will be the opposite, and so it must have opposite signs. The solution is $x = -15$.

Rule

When the isolated variable is negative, reverse the signs on all the terms to obtain the positive solution.

Caution: minus sign in front of a variable

Be careful to remember that a minus sign placed in front of a variable does not necessarily mean that the expression is negative. This is because the variable itself could be positive or negative. It is best to interpret this situation as "show the opposite of …" or "the negative of … ."

> *Example:* The value of "$-x$" would be negative if x is positive, and would positive if x is negative.

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PRACTICE

SOLUTIONS FOR SOLVING EQUATIONS BEGIN ON PAGE 137.

REINFORCEMENT PROBLEMS: SOLVING EQUATIONS

1. On a separate piece of paper, isolate the variable to solve each equation, and then check each solution. You may have to simplify some equations.

Equation	Answer
a. $x - 10 = 25$	
b. $x + 3 = 12$	
c. $x - 3 = -12$	
d. $3 - x = 15$	
e. $(-x - 8) = 32$	
f. $(-8x - 3) - (-7x + 1) = -49$	
g. $(15x - 8x) - (5x + x + 5) = 50$	
h. $-(-4x + 2) - 3x = 17$	

2. **True or false?** The expression "$-x$" represents a negative number. Explain your answer.

SOLUTIONS

PRACTICE QUESTIONS FOR SOLVING EQUATIONS BEGIN ON PAGE 136.

REINFORCEMENT PROBLEMS: SOLVING EQUATIONS

1.

Equation	Answer	Checking
a. $x - 10 = 25$	$x = 35$	$35 - 10 = 25$ ($25 = 25$, which is true)
b. $x + 3 = 12$	$x = 9$	$9 + 3 = 12$ ($12 = 12$, which is true)
c. $x - 3 = -12$	$x = -9$	$-9 - 3 = -12$ ($-12 = -12$, which is true)
d. $3 - x = 15$	$x = -12$	$3 - (-12) = 15$, which is $3 + 12 = 15$ ($15 = 15$, which is true)
e. $(-x - 8) = 32$	$x = -40$	$-(-40) - 8 = 32$, which is $40 - 8 = 32$ ($32 = 32$, which is true)
f. $(-8x - 3) - (-7x + 1) = -49$	$x = 45$	$(-8)(45) - 3 - [(-7)(45) + 1] = -49$, which is $-363 + 314 = -49$ (which is true)
g. $(15x - 8x) - (5x + x + 5) = 50$	$x = 55$	$[(15)(55) - (8)(55)] - [(5)(55) + 55 + 5] = 50$, which is $385 - 335 = 50$, which is true
h. $-(-4x + 2) - 3x = 13$	$x = 15$	$-[(-4)(15) + 2] - [(3)(15)] = 13$, which is $-(-58) - 45 = 13$, which is true

2. False. We do not know if x itself is negative or positive, so there is no way to know if the expression is negative or positive.