Essential Math for Accounting

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BRIEF OVERVIEW

Introduction

This section is designed to give you a review of all the math you will need for this book and for beginning your work in any introductory accounting text. Combined with the math review in the previous book of this series (Volume 1), you will have a review of all the math you will need for your entire first year of accounting study … and more.

How to use this section

You do not need to read the entire math review. Simply study those topics that you feel you need to practice.

- Read the topic that you feel you need to practice.
- When you finish reading, work the "Practice" problems for that topic.
- Review the solutions and highlight the problems you missed, so you can try them again or ask your instructor or classmates for more help.

Math review in the prior volume …

This is the second part of a two-part math review. The disk in Volume 1 contains the first math review, and includes:

- basic arithmetic operations
- rounding
- decimals
- percent
- positive and negative numbers
- how to evaluate an expression
- introduction to algebra

In this section, you will find:

▼ *Fractions: Parts of a Whole*

▼ *Multiplying Fractions*

▼ *Dividing Fractions*

Adding Fractions

Three Ways to Find the Lowest Common Denominator

Subtracting Fractions

Ratios

Averages

The Weighted Average

Continuation of Basic Algebra Review

Essential Terminology

Equations With a Variable On Only One Side

INTRODUCTION

A continuation of previous material . . .

This Essential Math for Accounting material is a continuation of the Essential Math for Accounting in the first book of this series. That math review covered the following topics:

- numerals
- the place-value system
- arithmetic operations
- decimals
- percent
- positive and negative numbers
- evaluating expressions
- introduction to algebra

Overview

The math review in this book is presented with the assumption that you understand and feel reasonably comfortable with the above topics, and covers the following areas:

- explanation and use of fractions
- averages
- ratios
- continuation of basic algebra topics

Fractions: Parts of a Whole

INTRODUCTION

Place-value system and whole numbersz

In the math review of the first book in this series, we began by studying how to express numerical amounts. First we looked at whole numbers like 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, etc. These amounts are equal to or greater than 1. We expressed these amounts as place-value numerals. In the **place-value** system, values are expressed as multiples of 10, according to where numerals are placed in relation to each other.

Place-value numbers between 0 and 1

After this, we began to examine small numbers—numbers that are between 0 and 1. We then decided that we could also use the place-value system to express these small amounts, too. In the place-value system, we show these small numbers as **decimals**, such as tenths, hundredths, and so on. We use a period mark to show a decimal. For example, the amount of two-tenths can be expressed as the decimal ".2"

Example

The example below shows you the number 2,879,533.572 with each of the individual place values identified. Notice how the decimal numerals are written to the right of the decimal point.

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones		Tenths	Hundredths	Thousandths
2,	8	7	9,	5	3	3	.	5	7	2

Percents—an alternative method

Later on in Volume 1, we also discovered that we could express all the same values that are shown in a place-value system in a different way—by using percents. In this approach, values are expressed as parts per 100, with 100 being an agreed-upon standard point of reference. We use the "%" symbol whenever we want to show that we are expressing values using the percent method. For example, expressing twenty-seven hundredths (.27) as a percent, we write "27%." Or, we can write 2.35 as "235%."

INTRODUCTION (continued)

Fractions—the third alternative

Fractions are simply a third alternative method for expressing exactly the same values that we can express using either the place-value system or the percent system.

Why fractions are different

A fraction is different because it expresses a value by showing some number of parts that come from a whole unit that consists of specified total number of parts. With a fraction:

- a value is expressed as some number of parts from a whole unit, and
- a whole unit can consist of any number of parts that we designate. This is very useful sometimes, and is different than the place-value system, which is based on using multiples of 10. It is also different than percents, which are based on expressions as parts per 100.

They all express the same values!

You should be clear that the place-value system, percents, and fractions are all just different ways of expressing the same values. You can always convert a value from one method to another.

So why use different methods?

Different methods are used because, in different situations, one method is sometimes easier to use or easier to understand than another method. Also, different people have different preferences. Therefore, you must be comfortable with all three methods of expressing values.

Example: alternative ways to express a value

Suppose that you wish to express the value twenty-five hundredths. You can:

- *Use decimals in the place-value numeral system:* This is .25.
- *Use percents:* In many cases, percents seem clearer to people. We can convert a decimal number to a percent, and say "twenty-five percent," written as "25%."
- *Use fractions:* A fraction would show this value as $\frac{25}{100}$, but could also use other number combinations. Next, we will study how fractions work.

INTRODUCTION (continued)

Special advantages of fractions

It is very useful to know how to use fractions because:

- many times a value will be expressed by comparing a given number of parts to the number of parts in one whole amount—such a comparison is a fraction.
- using fractions eliminates the need for rounding when calculating or expressing answers.
- it is the custom to express certain types of values as fractions.

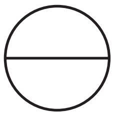
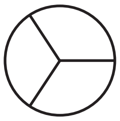
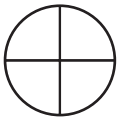
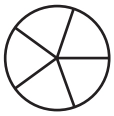
COUNTING THE PARTS IN A WHOLE AMOUNT

Overview

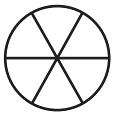
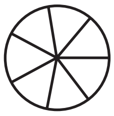
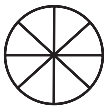
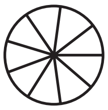
A fraction shows the number of equal-sized parts in the whole amount of something. This makes it possible for a fraction to express the whole amount of something by showing the whole as the total number of its parts.

Examples

In the following example, we are going to divide a pizza into equal parts. Therefore, the pizza is the whole amount which we will identify by its total parts.

If we divide a pizza into this many parts . . .	**we could visualize this . . .**	**and the total parts are expressed in English as . . .**
2		two-**halves**
3		three-**thirds**
4		four-**fourths**
5		five-**fifths**

COUNTING THE PARTS IN A WHOLE AMOUNT (continued)

If we divide a pizza into this many parts . . .	we could visualize this . . .	and the total parts are expressed in English as . . .
6		six-**sixths**
7		seven-**sevenths**
8		eight-**eighths**
9		nine-**ninths**

and so on . . .

We could keep dividing the pizza into smaller, equal-sized pieces. However, no matter how many parts we make, we can always express the whole amount of a pizza as the total number of its individual parts.

THE NAMES OF THE PARTS

Overview

What name to use to describe the parts in a whole amount depends upon the number of parts.

Names to use

The table on page 11 shows what names to use for some common equal-sized parts of a whole.

THE NAMES OF THE PARTS (continued)

If there are . . .	the parts are called . . .	If there are . . .	the parts are called . . .
2 parts	halves (each is a "half")	14 parts	fourteenths
3 parts	thirds	15 parts	fifteenths
4 parts	fourths	16 parts	sixteenths
5 parts	fifths	17 parts	seventeenths
6 parts	sixths	18 parts	eighteenths
7 parts	sevenths	19 parts	nineteenths
8 parts	eighths	20 parts	twentieths
9 parts	ninths	21 parts	twenty-firsts
10 parts	tenths	22 parts	twenty-seconds
11 parts	elevenths	23 parts	twenty-thirds
12 parts	twelfths	24 parts	twenty-fourths
13 parts	thirteenths	25 parts	twenty-fifths

General rules: for numbers 20 and above

General rules

- When a numeral ends in 1, replace the last digit with the word "firsts."

  Example: 41 parts would be called "forty-**firsts**."

- When a numeral ends in 2, replace the last digit with the word "seconds."

  Example: 72 parts would be called "seventy-**seconds**."

- When a numeral ends in 3 to 9, replace the name of the last digit with the name of the parts, as you see it done in the table above.

  Examples:
  — 175 parts would be "one hundred seventy-**fifths**."
  — 83 parts would be "eighty-**thirds**."

THE NAMES OF THE PARTS (continued)

General rules: for numbers 20 and above (continued)

- For 100 , 200, 300, or 1,000, 2,000, 3,000, or 10,000, etc., just add “ths” to the end of the word.

Example: 300 parts would be called “three hundred**ths**.”

Exceptions to the general rules

- For numbers 20, 30, 40, 50, 60, 70, 80, and 90, drop the “y” and add “ieths.”

Example:
— “eightieths”

EXPRESSING AN AMOUNT AS PART OF A WHOLE

Overview

At this point, you know how to identify the number of parts in a whole amount. Now you are ready to express any amount by using the number of parts in the whole. You will able to:

- show an amount less than the whole amount.
- show an amount equal to the whole amount.
- show an amount greater than the whole amount.

Procedure

The table on page 13 shows the procedure for expressing an amount by writing a fraction. You can imagine that we have a pizza that is divided into eight equal pieces, and we intend to eat two pieces. This is how we can use a fraction to show the amount that is two parts out of a whole amount of eight parts:

EXPRESSING AN AMOUNT AS PART OF A WHOLE (continued)

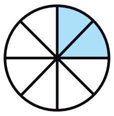
Step	Action	Example
1	Determine the number of equal parts in a single whole amount.	8
2	Determine the number of parts that you are comparing to the parts in a single whole amount.	2
3	Write the parts as a fraction:	
	a. Draw a horizontal line. Sometimes an angular line ("/") is also used.	———
	b. Write the total number of equal parts in the whole amount under the line. The number under the line is called the **denominator**.	$\frac{}{8} \longleftarrow$
	c. Above the line, write how many parts you are comparing to the parts in the whole amount. The number above the line is called the **numerator**.	$\longrightarrow \frac{2}{8}$

Showing less than a whole amount

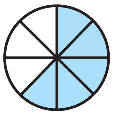
When the **numerator is less than the denominator**, the amount being expressed is always less than the whole amount.

Example: If the whole amount of a pizza is eight pieces, any numerator less than 8 always results in a fraction that is showing less than the whole amount, such as:

$\frac{2}{8}$, or $\frac{5}{8}$. You could visualize these amounts like this:



or

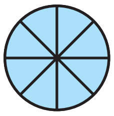


EXPRESSING AN AMOUNT AS PART OF A WHOLE (continued)

Showing exactly the whole amount

When the **numerator is the same as the denominator**, the fraction is showing a single whole amount.

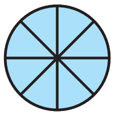
Example:
If the whole amount of a pizza is eight pieces, a numerator of 8 results in a fraction of $\frac{8}{8}$, which is one whole pizza. You could visualize this as:

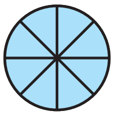


Showing more than the whole amount

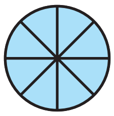
When the **numerator is greater than the denominator**, then the fraction is showing more than a single whole amount.

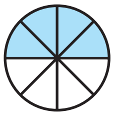
Example:
If the whole amount of a pizza is eight pieces, a numerator of 16 results in the fraction $\frac{16}{8}$, which expresses the equivalent of two whole pizzas. You could visualize this as:





Example:
If the whole amount of a pizza is eight pieces, a numerator of 12 results in the fraction $\frac{12}{8}$, which shows the equivalent of one whole pizza plus four parts—which is exactly half—of another pizza. You could visualize this as:





EXPRESSING AN AMOUNT AS PART OF A WHOLE (continued)

Writing the name of a fraction in English

To write a fraction amount in English:

- *Numerator:* Use the normal name for the numeral in the numerator.
- *Denominator:* Use the rules from page 11 to describe the name of the denominator.
- *Write the name of the numerator first.* Then write the name of the denominator.

> *Note:* Use a hyphen mark ("-") between the names of the numerator and the denominator if *both* the name of the numerator and the denominator are *not* hyphenated. (You *never* have to use a hyphen between the names if the denominator is 100 or more.)

Examples

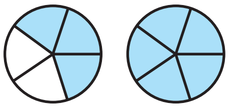
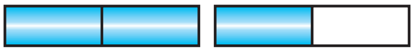
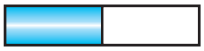
- $\frac{5}{10}$ is written as "five-tenths"
- $\frac{12}{7}$ is written as "twelve-sevenths"
- $\frac{1}{2}$ is written as "one-half"
- $\frac{3}{2}$ is written as "three-halves"
- $\frac{97}{200}$ is written as "ninety-seven two-hundredths"
- $\frac{44}{23}$ is written as "forty-four twenty-thirds"
- $\frac{11}{100}$ is written as "eleven hundredths"
- $\frac{19}{18}$ is written as "nineteen-eighteenths"
- $\frac{4}{1000}$ is written as "four thousandths"

PRACTICE

SOLUTIONS FOR WRITING FRACTIONS BEGIN ON PAGE 18.

REINFORCEMENT PROBLEMS: WRITING FRACTIONS

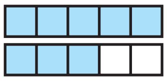
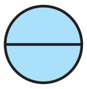
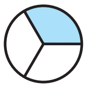
1. *Instructions:* For each amount described or illustrated, express the value as a fraction. The first two items are examples. Each figure is one whole amount divided into equal parts.

Description or Illustration	**Fraction**
a. 3 pieces from a pizza containing 5 equal pieces	$\frac{3}{5}$
b. 	$\frac{8}{5}$
c. 8 slices from a loaf of bread that has 20 equal slices	
d. 35 parts out of a total of 110 equal parts	
e. 20 parts from whole amounts that each contain 5 equal parts	
f. 	
g. 	
h. 17 marbles from a bag that contains 25 marbles	
i. 39 marbles compared to bags that each contain 25 marbles	
j. 	
k. A cash down payment of $500 on a total purchase of $1,750	
l. A collection of $90 from an account receivable of $90	
m. x parts from a whole that contains y equal parts	
n. z equal parts compared to a whole amount of $5z$ equal parts	
o. x from a whole amount of $4(y + 1)^2$	
p. y compared to a whole amount of $5y$	

PRACTICE

SOLUTIONS FOR WRITING FRACTIONS BEGIN ON PAGE 18.

2. *Instructions:* For each amount written as a fraction or illustrated, write the value in English. Use the first item as an example. Each figure represents one whole amount divided into equal parts.

Fraction or Illustration	**Written as …**
a. $\frac{3}{4}$	three-fourths
b. $\frac{11}{30}$	
c. $\frac{11}{37}$	
d. 	
e. $\frac{147}{100}$	
f. $\frac{750}{1000}$	
g. 	
h. $\frac{1}{2}$	
i. $\frac{12}{3}$	
j. $\frac{10}{17}$	
k. $\frac{1}{3}$	
l. 	
m. $\frac{7}{6}$	
n. $\frac{1}{100}$	

PRACTICE

3. *Instructions.* For each fraction described in English, write the fractional value in numerals.

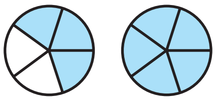
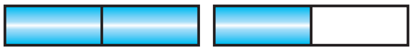
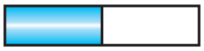
Description	Fraction
a. ninety-three two-hundredths	
b. seven-eighths	
c. nineteen forty-sevenths	
d. twelve two hundred-firsts	
e. eleven-eighteenths	
f. five-halves	
g. one-seventh	
h. five ten-thousandths	
i. seventy-seven hundred-thousandths	
j. forty-four twelfths	
k. fifteen-thirds	

SOLUTIONS

PRACTICE QUESTIONS FOR WRITING FRACTIONS BEGIN ON PAGE 16.

REINFORCEMENT PROBLEMS: WRITING FRACTIONS

1.

Description or Illustration	Fraction
a. 3 pieces from a pizza containing 5 equal pieces	$\frac{3}{5}$
b. 	$\frac{8}{5}$
c. 8 slices from a loaf of bread that has 20 equal slices	$\frac{8}{20}$
d. 35 parts out of a total of 110 equal parts	$\frac{35}{110}$
e. 20 parts from whole amounts that each contain 5 equal parts	$\frac{20}{5}$
f. 	$\frac{3}{2}$
g. 	$\frac{1}{2}$

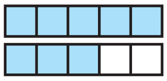
SOLUTIONS

PRACTICE QUESTIONS FOR WRITING FRACTIONS BEGIN ON PAGE 16.

1, *continued*

Description or Illustration	**Fraction**
h. 17 marbles from a bag that contains 25 marbles	$\frac{17}{25}$
i. 39 marbles compared to bags that each contain 25 marbles	$\frac{39}{25}$
j. 	$\frac{8}{3}$
k. A cash down payment of \$500 on a total purchase of \$1,750	$\frac{500}{1750}$
l. A collection of \$90 from an account receivable of \$90	$\frac{90}{90}$
m. x parts from a whole that contains y equal parts	$\frac{x}{y}$
n. z equal parts compared to a whole amount of $5z$ equal parts	$\frac{z}{5z}$
o. x from a whole amount of $4(y + 1)^2$	$\frac{x}{4(y+1)^2}$
p. y compared to a whole amount of $5y$	$\frac{y}{5y}$

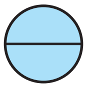
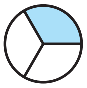
2.

Fraction or Illustration	**Written as …**
a. $\frac{3}{4}$	three-fourths
b. $\frac{11}{30}$	eleven-thirtieths
c. $\frac{11}{37}$	eleven thirty-sevenths
d. 	eight-fifths
e. $\frac{147}{100}$	one hundred forty-seven hundredths

SOLUTIONS

PRACTICE QUESTIONS FOR WRITING FRACTIONS BEGIN ON PAGE 16.

2, *continued*

Fraction or Illustration	**Written as …**
f. $\frac{750}{1000}$	seven hundred fifty thousandths
g. 	two-halves
h. $\frac{1}{2}$	one-half
i. $\frac{12}{3}$	twelve-thirds
j. $\frac{10}{17}$	ten-seventeenths
k. $\frac{1}{3}$	one-third
l. 	one-third
m. $\frac{7}{6}$	seven-sixths
n. $\frac{1}{100}$	one-hundredth

3.

Description	**Fraction**
a. ninety-three two hundredths	93/200
b. seven-eighths	7/8
c. nineteen forty-sevenths	19/47
d. twelve two hundred-firsts	12/201
e. eleven-eighteenths	11/18
f. five-halves	5/2
g. one-seventh	1/7
h. five ten-thousandths	5/10,000
i. seventy-seven hundred-thousandths	77/100,000
j. forty-four twelfths	44/12
k. fifteen-thirds	15/3

MIXED NUMERALS

Definition

A mixed numeral is a number value that is expressed by using both a whole numeral and a fraction.

Example: $2\frac{1}{4}$

How to express in English

- Say the whole numeral first, as you normally say it.
- Then use "and."
- Then say the fraction as you normally would.

Example: $2\frac{1}{4}$ is expressed as "two and one-fourth."

Why mixed numerals are used

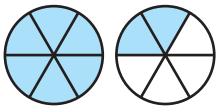
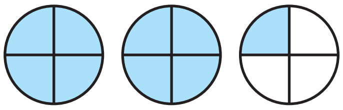
A mixed numeral is used because it is:

- a quick way of showing the number of whole amounts of something plus any "leftover" that is less than a whole amount.
- an alternative to expressing the same value only as a fraction.

> *Note:* A mixed numeral is used only when there are one or more whole amounts to show.

Examples

The following table shows how a mixed numeral can be used.

Description/Illustration	As a Fraction	As a Mixed Numeral
8 pieces from 2 pizzas that have 6 pieces each: 	$\frac{8}{6}$	$1\frac{2}{6}$ (1 whole pizza plus 2 of the 6 pieces of another)
9 pieces from 3 pizzas that have 4 pieces each: 	$\frac{9}{4}$	$2\frac{1}{4}$ (2 whole pizzas plus 1 of the 4 pieces of another)

HOW TO WRITE A FRACTION AS A MIXED (OR WHOLE) NUMERAL

Overview

It is common practice to rewrite a fraction into a mixed numeral or a whole numeral. This is done for fractions that have numerators that are bigger than the denominators. These kinds of fractions are called **improper fractions**.

Procedure

The table below shows how to express a fraction as a mixed or whole numeral.

<table>
<tr><th>Step</th><th colspan="2">Action</th><th>Example</th></tr>
<tr><td>1</td><td colspan="2">Identify a fraction that has a numerator which is bigger than the denominator (a fraction greater than 1).</td><td>Improper fraction → $\frac{74}{5}$</td></tr>
<tr><td>2</td><td colspan="2">Divide the denominator into the numerator.</td><td>$\begin{array}{r} 14 \\ 5\overline{)74} \\ \underline{5} \\ 24 \\ \underline{20} \\ 4 \end{array}$
Quotient: 14 with remainder of 4</td></tr>
<tr><td rowspan="3">3</td><td>IF the quotient …</td><td>THEN …</td><td></td></tr>
<tr><td>is a whole number with no remainder,</td><td>the whole number is the answer.</td><td></td></tr>
<tr><td>contains a remainder,</td><td>the whole number from the quotient is used in the mixed numeral as the whole numeral
The fraction part of the mixed numeral uses the:
• remainder as the numerator
• divisor as the denominator</td><td>$14\frac{4}{5}$</td></tr>
</table>

Note: The answer would be expressed in English as "fourteen and four-fifths."

HOW TO WRITE A MIXED NUMERAL AS A FRACTION

Procedure

Sometimes it is necessary to convert a mixed numeral into an improper fraction. The table below shows how to do this.

Step	Action	Example
1	Identify the mixed numeral you wish to express as an improper fraction.	$14\frac{4}{5}$
2	Multiply the whole numeral by the denominator of the fraction.	$14 \times 5 = 70$
3	Add the numerator of the fraction to the product from **STEP 2.**	$\begin{array}{r} 70 \\ +\ \ 4 \\ \hline 74 \end{array}$
4	● The numerator of the fraction is the total from **STEP 3.** ● The denominator is the same denominator that is in the mixed numeral.	$\frac{74}{5}$

Note: The answer would be expressed in English as "seventy-four fifths."

What they have in common

Whether you choose to express a value as a fraction or as a mixed numeral, notice that they both have the same feature: *the denominator always shows how many parts in one whole unit.*

PRACTICE

SOLUTIONS FOR MIXED NUMERALS BEGIN ON PAGE 25.

REINFORCEMENT PROBLEMS: MIXED NUMERALS

1. *Instructions:* For each amount described, express the value as a mixed numeral.

Description	Mixed Numeral
a. Seven and two-thirds	
b. One and five-eighths	
c. Two hundred three and ninety-nine one-hundredths	
d. Fourteen and seven-elevenths	
e. Forty-six and three twenty-seconds	

---

2. *Instructions:* In the "Item" column in the table below, you will see either a mixed numeral or a fraction. If the item is a mixed numeral, convert it to an improper fraction. If the item is an improper fraction, convert it to a mixed numeral. Write your answer in the "Answer" column next to the item you are converting. If it is not possible to do a conversion, write "not possible."

Item	Answer	Item	Answer
a. $4\frac{7}{10}$		k. $12\frac{2}{3}$	
b. $2\frac{1}{3}$		l. $\frac{38}{3}$	
c. $\frac{15}{10}$		m. $2\frac{3}{5}$	
d. $\frac{34}{12}$		n. $\frac{13}{5}$	
e. $9\frac{1}{30}$		o. $\frac{21}{45}$	
f. $\frac{300}{52}$		p. $\frac{12}{5}$	
g. $\frac{28}{28}$		q. $\frac{89}{8}$	
h. $\frac{4}{1}$		r. $\frac{700}{500}$	
i. $10\frac{21}{45}$		s. $14\frac{5}{17}$	
j. $\frac{1}{4}$		t. $x\frac{y}{z}$	

SOLUTIONS

PRACTICE QUESTIONS FOR MIXED NUMERALS BEGIN ON PAGE 24.

REINFORCEMENT PROBLEMS: MIXED NUMERALS

1.

Description	Mixed Numeral
a. Seven and two-thirds	$7\frac{2}{3}$
b. One and five-eighths	$1\frac{5}{8}$
c. Two hundred three and ninety-nine one-hundredths	$203\frac{99}{100}$
d. Fourteen and seven-elevenths	$14\frac{7}{11}$
e. Forty-six and three twenty-seconds	$46\frac{3}{22}$

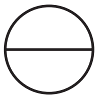
2.

Item	Answer	Item	Answer
a. $4\frac{7}{10}$	$\frac{47}{10}$	k. $12\frac{2}{3}$	$\frac{38}{3}$
b. $2\frac{1}{3}$	$\frac{7}{3}$	l. $\frac{38}{3}$	$12\frac{2}{3}$
c. $\frac{15}{10}$	$1\frac{5}{10}$	m. $2\frac{3}{5}$	$\frac{13}{5}$
d. $\frac{34}{12}$	$2\frac{10}{12}$	n. $\frac{13}{5}$	$2\frac{3}{5}$
e. $9\frac{1}{30}$	$\frac{271}{30}$	o. $\frac{21}{45}$	not possible—number is less than 1
f. $\frac{300}{52}$	$5\frac{40}{52}$	p. $\frac{12}{5}$	$2\frac{2}{5}$
g. $\frac{28}{28}$	1	q. $\frac{89}{8}$	$11\frac{1}{8}$
h. $\frac{4}{1}$	4	r. $\frac{700}{500}$	$1\frac{200}{500}$
i. $10\frac{21}{45}$	$\frac{471}{45}$	s. $14\frac{5}{17}$	$\frac{243}{17}$
j. $\frac{1}{4}$	not possible—number is less than one 1	t. $x\frac{y}{z}$	$\frac{(x\ z)+y}{z}$

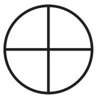
CONVERT A WHOLE NUMBER TO A FRACTION

Determine the size of each part

- Suppose that you buy a pizza and divide it into **two pieces**. The pizza would look like this:

$1 = \frac{2}{2}$  so one equals **two-halves**, and each part is $\frac{1}{2}$

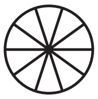
- You could also divide the same whole pizza into **four pieces**. The pizza would look like this:

$1 = \frac{4}{4}$  so one equals **four-fourths**, and each part is $\frac{1}{4}$

- You could also divide the same whole pizza into **six pieces**. The pizza would look like this:

$1 = \frac{6}{6}$  so one equals **six-sixths**, and each part is $\frac{1}{6}$

- You could also divide the same whole pizza into **10 pieces**. The pizza would look like this:

$1 = \frac{10}{10}$  so one equals **ten-tenths**, and each part is $\frac{1}{10}$

Rule: You can convert a whole amount into a fraction of any number of equal parts. Decide on the number of parts in the whole, and show that amount of parts in both the numerator and denominator.

EQUIVALENT FRACTIONS

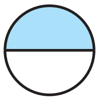
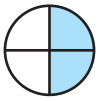
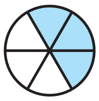
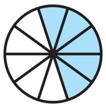
Overview

The same portion of one whole amount can be expressed by many fractions. This will happen when the same whole amount is divided into different numbers of equal parts.

EQUIVALENT FRACTIONS (continued)

Example

In the above discussion about single parts of a whole amount, you saw the example of one whole pizza being divided into two pieces, four pieces, six pieces, and 10 pieces. Now suppose that we wanted to show that half of a pizza had been eaten. How would we show that?

- For the two-piece pizza, half the pizza  would be the fraction $\frac{1}{2}$
- For the four-piece pizza, half the pizza  would be the fraction $\frac{2}{4}$
- For the six-piece pizza, half the pizza  would be the fraction $\frac{3}{6}$
- For the 10-piece pizza, half the pizza  would be the fraction $\frac{5}{10}$

Conclusion: The fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{3}{6}$, and $\frac{5}{10}$ are all equivalent to each other. They are just different ways of expressing half of the same whole amount, if it is divided into a different number of equal parts.

RAISING A FRACTION TO HIGHER TERMS

Overview

Situations sometimes arise when it is more useful to show the same total value of a fraction, but to do it by using a fraction in which the whole has more parts. Although the new fraction has more parts, it is equivalent to the old fraction.

Definition

Converting a fraction into a different but *equivalent fraction* with more parts is called **raising a fraction to higher terms**.

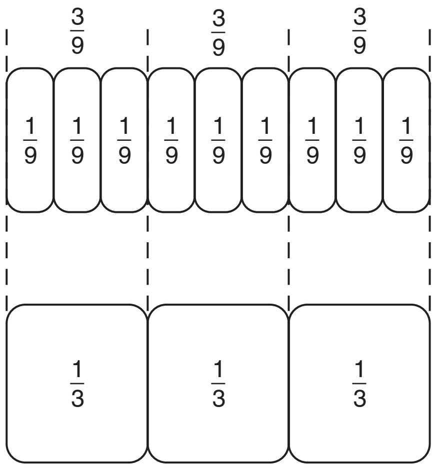
More parts

Higher term fractions always have a larger numerator and denominator than the original fraction.

RAISING A FRACTION TO HIGHER TERMS (continued)

Example

Suppose that you bake a loaf of fresh bread, which you plan to serve to three guests at dinner. This means that each guest will receive one-third of the loaf. However, to your surprise, each guest brings two friends. This means that the whole loaf will now have to be divided among nine people. So, each one-third of the loaf will now have to be shared by three people. There are now more parts than before. How is each one-third to be divided now? The diagram below illustrates the new portions.



Now: 9 people share

$\frac{9}{9}$ = 1 whole loaf

Before: 3 people share

$\frac{3}{3}$ = 1 whole loaf

Because of the new situation, each $\frac{1}{3}$ has been raised to a new fraction of higher terms: $\frac{3}{9}$. However, the $\frac{3}{9}$ is still exactly equivalent to $\frac{1}{3}$ of the loaf.

Rule: raising to higher terms

To raise a fraction to higher terms, multiply *both* the numerator and the denominator of a fraction by the *same* nonzero number.

Value not changed

Multiplying both the numerator and denominator in the fraction $\frac{1}{3}$

by the same amount of 3 *does not change the value of the fraction*.

So, $\frac{1\times 3}{3\times 3}$ results in $\frac{3}{9}$

which is really the same total value, just expressed by *using a fraction with more parts*. Each one-third portion is now expressed as three-ninths.

RAISING A FRACTION TO HIGHER TERMS (continued)

What should I multiply by?

In the example on page 28, we multiplied the fraction by 3, because each portion had to be separated into three times more pieces. If we had wanted to separate each portion into five times more pieces, we would have multiplied the numerator and denominator by 5, and raised the fraction to 5/15. For a portion that is seven times more pieces, we would multiply by 7 and raise the fraction to 7/21, and so on.

Procedure: when you know how many times more parts

When you know the factor of how many times more total parts are needed (the denominator) or the factor of how many times more each portion (the numerator) must be, multiply the numerator and denominator by that factor.

Examples

- Raise $\frac{2}{5}$ to an equivalent fraction that has five times more total parts.

  Answer: Multiply by 5 $\frac{2 \cdot 5}{5 \cdot 5} = \frac{10}{25}$
- Raise $\frac{4}{7}$ to an equivalent fraction with portions of four times more parts.

  Answer: Multiply by 4 $\frac{4 \cdot 4}{7 \cdot 4} = \frac{16}{28}$

Procedure: when you only know the new numerator or denominator

Sometimes only the numerator or the denominator of the new fraction will be known to you. However, from this you can determine the correct multiple.

The table below and continued on page 30 shows how to use the numerator or the denominator to determine the correct multiple.

If …	Then …	Example
You know the numerator of the new fraction	• Divide the numerator of the old fraction into the numerator of the new fraction. • Use the result as the amount by which to multiply the old fraction.	Raise the fraction 3/4 to an equivalent fraction of higher terms which has a numerator of 15. *Answer*: • $15 \div 3 = 5$ • $\frac{3 \cdot \mathbf{5}}{4 \cdot \mathbf{5}} = \frac{15}{20}$

RAISING A FRACTION TO HIGHER TERMS (continued)

Procedure: when you only know the new numerator or denominator (continued)

If …	Then …	Example
You know the denominator of the new fraction	- Divide the denominator of the old fraction into the denominator of the new fraction. - Use the result as the amount by which to multiply the old fraction.	1. Raise the fraction 2/7 to an equivalent fraction of higher terms which has a denominator (total parts) of 84. - $84 \div 7 = 12$ - $\frac{2\cdot\mathbf{12}}{7\cdot\mathbf{12}}=\frac{24}{84}$ 2. Raise the fraction 2/7 to an equivalent fraction of twenty-eighths. - $28 \div 7 = 4$ - $\frac{2\cdot\mathbf{4}}{7\cdot\mathbf{4}}=\frac{4}{28}$

More examples

- $\frac{2}{3}=\frac{?}{6}$ *Answer:* $\frac{2\cdot2}{3\cdot2}=\frac{4}{6}$	- $\frac{4}{9}=\frac{?}{45}$ *Answer:* $\frac{4\cdot5}{9\cdot5}=\frac{20}{45}$
- $\frac{1}{5}=\frac{8}{?}$ *Answer:* $\frac{1\cdot8}{5\cdot8}=\frac{8}{40}$	- $\frac{3}{4}=\frac{9}{?}$ *Answer:* $\frac{3\cdot3}{4\cdot3}=\frac{9}{12}$

PRACTICE

REINFORCEMENT PROBLEMS: RAISING FRACTIONS TO HIGHER TERMS

Instructions: Based on the information given to you about the new fraction, raise the old fraction to higher terms. Write your answer in the space provided.

Fraction	New Fraction	Answer	Fraction	New Fraction	Answer
1. 2/6	has 4 times more total parts		11. 2/ 5	parts are hundredths	
2. 2/6	parts are twenty-fourths		12. 2/5	parts are thousandths	
3. 5/8	15/?		13. 2/5	parts are eightieths	
4. 19/13	?/65		14. 23/36	?/108	
5. 10/12	numerator is 3 times greater		15. 2/11	has 7 times more total parts	
6. 12/9	parts are thirty-sixths		16. 21/15	?/45	
7. 2/9	?/27		17. $x/7$	parts are twenty-firsts	
8. 14/43	?/86		18. $15/2x$	$?/18x$	
9. 5/8	40/?		19. x/y	$4x/?$	
10. 7/11	each portion has 5 times more parts in it		20. $(x – 1)/y$	$?/3y$	

SOLUTIONS

Fraction	New Fraction	Answer	Fraction	New Fraction	Answer
1. 2/6	has 4 times more total parts	8/24	11. 2/5	parts are hundredths	40/100
2. 2/6	parts are twenty-fourths	8/24	12. 2/5	parts are thousandths	400/1,000
3. 5/8	15/?	15/24	13. 2/5	parts are eightieths	32/80
4. 19/13	?/65	95/65	14. 23/36	?/108	69/108
5. 10/12	numerator is 3 times greater	30/36	15. 2/11	has 7 times more total parts	14/77
6. 12/9	parts are thirty-sixths	48/36	16. 21/15	?/45	63/45
7. 2/9	?/27	6/27	17. $x/7$	parts are twenty-firsts	$3x/21$
8. 14/43	?/86	28/86	18. $15/2x$	$?/18x$	$135/18x$
9. 5/8	40/?	40/64	19. x/y	$4x/?$	$4x/4y$
10. 7/11	each portion has 5 times more parts in it	35/55	20. $(x – 1)/y$	$?/3y$	$3(x – 1)/3y$

LOWER AND LOWEST TERMS

Overview

Just as it is sometimes necessary to raise a fraction to higher terms, it is also sometimes necessary to reduce a fraction to an equivalent fraction of lower terms. Normally a fraction is reduced to lower terms in order to express the fraction more clearly. The clearest way to express a fraction is by converting it into *lowest* terms.

Definition: lower terms

Reducing a fraction to lower terms means converting a fraction into an equivalent faction which has fewer parts.

Examples

Fraction	Lower term equivalent fraction
24/36	12/18
150/240	15/24
120/200	60/100

Definition: lowest terms

Reducing a fraction to lowest terms means finding an equivalent fraction which:

- has the smallest possible whole number denominator, and
- the numerator and denominator cannot be evenly divided by the same number, except 1.

Examples

Fraction	Lowest term equivalent fraction
24/36	2/3
150/240	5/8
120/200	3/5

HOW TO REDUCE TO LOWER TERMS

Method

To reduce a fraction to lower terms, divide both the numerator and denominator by the same whole number, except 1, that leaves no remainder.

HOW TO REDUCE TO LOWER TERMS (continued)

Example

- Reduce $\frac{20}{30}$ to lower terms: $\frac{20 \div 5}{30 \div 5} = \frac{4}{6}$

HOW TO REDUCE TO LOWEST TERMS

Overview

Generally, it is more important to be able to reduce a fraction to the lowest terms possible. Fractional answers to problems are usually shown in lowest terms. This always results in the most easy-to-understand fraction.

Trial and error method

The table below shows the steps to use in the trial-and-error method. (The example reduces the fraction 120/200 to its lowest terms of 3/5.)

Step	Action	Example
1	Examine the numerator and the denominator until you find any whole number that will divide evenly into both of them.	Reduce the fraction $\frac{120}{200}$ to lowest terms: • Both 120 and 200 can be evenly divided by 10.
2	Divide both the numerator and denominator by that whole number, and use the quotients as as a new fraction.	$\frac{120 \div 10}{200 \div 10} = \frac{12}{20}$
3	Repeat **STEPS 1** and **2** as many times as necessary until no more whole numbers will divide evenly into the numerator and denominator, except 1.	$\frac{12 \div 4}{20 \div 4} = \frac{3}{5}$ • Both 12 and 20 can be evenly divided by 4. The fraction cannot be reduced any lower.

Greatest common divisor (GCD)

The **greatest common divisor** is a number that will immediately reduce a fraction to its lowest terms, without needing to use trial and error.

HOW TO REDUCE TO LOWER TERMS (continued)

GCD method for reducing fractions

1. Find the greatest common divisor.
2. Divide it into both the numerator and denominator.
3. Use the quotients as the new fraction.

How to find a GCD

The table below shows the steps to find a greatest common divisor. (The example finds the greatest common divisor for 120/200.)

Step	Action	Example
1	Examine the numerator and denominator. Divide the smaller number into the larger number.	To find the GCD of $\frac{120}{200}$, divide: $\begin{array}{r}1\\120\overline{)200}\end{array}$ remainder of **80**
2	If there is a remainder, divide it into the divisor.	$\begin{array}{r}1\\80\overline{)200}\end{array}$ remainder of **40**
3	Continue dividing each new remainder into the divisor until you obtain a division for which there is no remainder.	$\begin{array}{r}2\\40\overline{)80}\end{array}$ no remainder
4	The final divisor (here, 40) is the greatest common divisor.	$\frac{120 \div 40}{200 \div 40} = \frac{3}{5}$

Not every fraction can be reduced

Some fractions, such as 37/39, have a greatest common divisor of only 1. This means that the fraction is already in the lowest possible terms. Sometimes this is obvious, but frequently you will have to go through the steps to find out.

PRACTICE

REINFORCEMENT PROBLEMS: REDUCING FRACTIONS TO LOWEST TERMS

1. *Instructions:* Reduce each of the fractions shown below to its lowest terms. Write your answer in the space provided next to each fraction. **Use both methods**.

Fraction	Lowest Terms	Fraction	Lowest Terms
a. 12/60		h. 70/130	
b. 24/40		i. 9/12	
c. 14/112		j. 54/126	
d. 42/84		k. 78/96	
e. 91/156		l. 10/35	
f. 5/70		m. 6/27	
g. 85/306		n. 204/210	

2. *Review:* What is the difference between raising a fraction to higher terms and reducing a fraction to lowest terms? Why are these procedures done?

SOLUTIONS

1.

Fraction	Lowest Terms	Fraction	Lowest Terms
a. 12/60	1/5	h. 70/130	7/13
b. 24/40	3/5	i. 9/12	3/4
c. 14/112	1/8	j. 54/126	3/7
d. 42/84	1/2	k. 78/96	13/16
e. 91/156	7/12	l. 10/35	2/7
f. 5/70	1/14	m. 6/27	2/9
g. 85/306	5/18	n. 204/210	34/35

2. Each procedure is done in order to express a fraction so it is either easier to use in a calculation or easier to understand as an answer. This will depend on each situation. Answers that are fractions are almost always reduced to lowest terms, because smaller fractions are easier to understand.

   Raising a fraction to higher terms means multiplying both the numerator and denominator by the same number, so both the numerator and denominator in the new fraction are bigger. However, the resulting fraction is *still equivalent in value to the original*.

   Reducing a fraction to lowest terms means dividing the numerator and denominator by the same number, so both the numerator and denominator are reduced to the point where each can be evenly divided only by 1. However, the resulting fraction is *still equivalent in value to the original.*

CONVERTING FRACTIONS INTO DECIMALS AND INTO PERCENT

Overview

The need to convert a fraction into a decimal or a percent form is a common occurrence. This is especially true for numbers that are less than one.

Although all three forms (decimal, percent, and fraction) can express exactly the same value, sometimes it is more convenient to work with numbers in one form than another. This situation may arise because of calculations you are doing, or it may simply involve writing a report when you know that the readers prefer and expect to see numbers expressed in a certain way.

Procedure

The table below shows how to convert fractions into decimals and percents.

Step	Action	Example
1	Select a fraction and decide how many places to the right of the decimal point you want to express it.	Convert 3/8 to a decimal, expressing the answer to the thousandths place (3 places to the right of the decimal point). Then express the number as a percent.
2	Divide the denominator into the numerator, until the quotient is *one place more* than the required number of places.	$\begin{array}{r} .3750 \\ 8\overline{)3.08000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 0 \end{array}$ **Quotient to four places**
3	If necessary, round the decimal to the desired place, using the rules for rounding. (See Volume 1.)	In this case, you do not need to apply the rules for rounding. .3750 is simply .375
4	To convert the decimal to a percent, multiply by 100, and write the “%” symbol.	.375 × 100 = 37.5%

Same value

Notice in the example above that 3/8, .375, and 37.5% all express exactly the same value. Only the form of expression is different.

CONVERTING FRACTIONS INTO DECIMALS AND INTO PERCENT (continued)

Nonterminating quotients

A **nonterminating quotient** is a quotient that never ends. You can continue dividing, and there will always be a remainder. These kinds of answers always need to be rounded to a specified number of places. The table below shows you an example.

Step	Action	Example
1	Select a fraction and decide how many places to the right of the decimal point you want to express it.	Convert 3/7 to a decimal. Your instructor requires decimal accuracy to thousandths place.
2	Divide the denominator into the numerator, until the quotient is *one place more* than the required number of places. *Note:* Notice that the quotient is nonterminating.	$\begin{array}{r} .4285 \\ 7\overline{)3.0000} \\ \underline{28} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ 35 \end{array}$ **Quotient to four places**
3	Round the decimal to the desired place, using the rules for rounding. (See Volume 1.)	.4285 rounded is .429
4	To convert the decimal to a percent, multiply by 100, and write the "%" symbol.	The problem did not ask for conversion to a percent. However, the percent is 42.9%.

What if I have a mixed numeral?

If you have a mixed numeral, convert it to a fraction and then follow the procedure above. The method for converting a mixed numeral to a fraction is on page 23.

CONVERTING DECIMALS INTO FRACTIONS

Overview

Converting a decimal into a fraction is the reverse of converting a fraction to a decimal, but is usually required for similar reasons—to facilitate certain calculations or to express values in a desired way.

Procedure for decimal number less than 1

The table below shows you the procedure for converting a decimal that is less than 1 into a fraction.

Step	Action	Example
1	Determine the number of places to the right of the decimal point.	Convert .375 to a fraction. ● Places to right of decimal: 3 ● Expressed as: thousandths
2	Remove the decimal point and use the number as a numerator above a fraction bar.	$\frac{375}{}$
3	Write in a denominator that corresponds to the places expressed in **STEP 1.**	$\frac{375}{1000}$
4	Find the greatest common divisor (GCD) and reduce the fraction to lowest terms. *Note:* GCD method is defined on page 33.	GCD is 125. $\frac{375 \div 125}{1000 \div 125} = \frac{3}{8}$

Another example

Convert .5255 into a fraction:

1. Places to right of decimal: 4 (expressed as: ten thousandths)
2. Numerator is: $\frac{5255}{}$
3. Writing in the denominator, the fraction is: $\frac{5255}{10000}$
4. Reduced to lowest terms: $\frac{1051}{2000}$

CONVERTING DECIMALS INTO FRACTIONS (continued)

Procedure for decimal number greater than 1

The procedure for converting a decimal number greater than 1 to a fraction is the same as you just learned, with one added step.

The final step is to express the answer as a mixed numeral (see page 21) by identifying the whole number that is to the left of the decimal point, and then writing it to the left of the fraction obtained in Step 4 (see page 38).

The mixed numeral can then be expressed as an improper fraction, if desired.

Example

Express the amount of 4.375 as a fraction.

Steps:

1. Places to right of decimal: 3 (expressed as: thousandths)
2. Numerator is: $\frac{375}{}$
3. Writing in the denominator, the fraction is: $\frac{375}{1000}$
4. Reduced to lowest terms: $\frac{3}{8}$
5. Making a mixed numeral, the amount is: $4\frac{3}{8}$

   which can be converted to $\frac{35}{8}$

PRACTICE

SOLUTIONS FOR CONVERTING FRACTIONS AND DECIMALS BEGIN ON PAGE 41.

REINFORCEMENT PROBLEMS: CONVERTING FRACTIONS AND DECIMALS

1. *Instructions:* Convert the fractions and mixed numerals in the table into decimals. Write your answers in the spaces provided. Round to three decimal places, if necessary.

Fraction	Answer	Fraction	Answer
a. $\frac{1}{4}$		f. $\frac{36}{9}$	
b. $\frac{1}{12}$		g. $\frac{485}{1000}$	
c. $\frac{1}{2}$		h. $\frac{485}{100}$	
d. $\frac{3}{2}$		i. $4\frac{7}{9}$	
e. $\frac{17}{25}$		j. $35\frac{8}{10}$	

2. *Instructions:* Convert the decimals in the table into fractions. Write your answers in the spaces provided.

Decimal	Answer	Decimal	Answer
a. .25		f. .72	
b. 1.5		g. .535	
c. 7.38		h. 2.88	
d. .0852		i. .4	
e. .0041		j. .875	

PRACTICE

3. *Instructions:* Stock prices per share traditionally were quoted as mixed numerals, but this has changed in most cases to decimal dollar amounts. Convert the stock price shown as a mixed numeral to a dollar amount. Carry the dollar amount to as many places as needed to the right of the decimal point that results in no remainder. Use the first item as an example.

Stock Name	**Stock Quote**	**Dollar Price**
Novell	$34\frac{5}{16}$	$34.3125
Cisco	$104\frac{7}{16}$	
IBM	$108\frac{5}{8}$	
Analogy	$2\frac{5}{32}$	
Apple Computer	$103\frac{1}{2}$	
United Health Care	$50\frac{121}{128}$	

SOLUTIONS

PRACTICE QUESTIONS FOR CONVERTING FRACTIONS AND DECIMALS BEGIN ON PAGE 40.

REINFORCEMENT PROBLEMS: CONVERTING FRACTIONS AND DECIMALS

1. a. .25
   b. .083 (rounded)
   c. .5
   d. 1.5
   e. .68
   f. 4
   g. .485
   h. 4.85
   i. 4.778 (rounded)
   j. 35.8

2. a. $\frac{1}{4}$
   b. $\frac{3}{2}$
   c. $7\frac{19}{50}$ or $\frac{369}{50}$
   d. $\frac{213}{2500}$
   e. $\frac{41}{10{,}000}$
   f. $\frac{18}{25}$
   g. $\frac{107}{200}$
   h. $2\frac{22}{25}$ or $\frac{72}{25}$
   i. $\frac{2}{5}$
   j. $\frac{7}{8}$

3. Dollar prices are:

   Novell: $34.3125
   Cisco: $104.4375
   IBM: $108.625
   Analogy: $2.15625
   Apple Computer: $103.50
   United Health Care: $50.9453125

HOW TO COMPARE THE SIZE OF TWO FRACTIONS

Procedure

The table below shows the procedure for comparing the size of fractions.

<table>
<tr><th>Step</th><th>Action</th></tr>
<tr><td>1</td><td>● If the fractions have different denominators, go to STEP 2,
● Otherwise, the fraction with largest numerator is the largest number.</td></tr>
<tr><td>2</td><td>If the fractions have different denominators, you must find a common denominator of both fractions.
<table>
<tr><th>ACTION</th><th>EXAMPLE</th></tr>
<tr><td>Identify the denominators of the two fractions you are comparing.</td><td>Which is larger,
$\frac{3}{4}$ or $\frac{4}{7}$?
The denominators are 4 and 7.</td></tr>
<tr><td>Multiply each fraction by the denominator of the other fraction.</td><td>$\frac{3 \cdot \mathbf{7}}{4 \cdot \mathbf{7}} = \frac{21}{28}$ and $\frac{4 \cdot \mathbf{4}}{7 \cdot \mathbf{4}} = \frac{16}{28}$</td></tr>
</table>
</td></tr>
<tr><td>3</td><td>Compare the numerators of the two new fractions.
<table>
<tr><th>IF …</th><th>THEN …</th></tr>
<tr><td>the numerators are different,</td><td>the fraction with the larger numerator is the larger fraction.</td></tr>
<tr><td>the numerators are the same,</td><td>the fractions are equal.</td></tr>
</table>
(Here, 21 is larger than 16, so $\frac{3}{4}$ is the larger fraction.)
<i>Note:</i> You can also convert the fractions into decimals and compare the size of the decimals.</td></tr>
</table>

Mixed numerals

To compare mixed numerals, simply convert each mixed numeral to a fraction, and then follow the procedure shown above.

HOW TO COMPARE THE SIZE OF TWO FRACTIONS (continued)

Example

Compare $3\frac{4}{12}$ to $2\frac{24}{15}$

Converting these to fractions (see page 21 for procedure), we obtain the fractions $\frac{40}{12}$ and $\frac{54}{15}$

then just follow the procedure above. The resulting fractions are

$\frac{600}{180}$ and $\frac{648}{180}$

so, $2\frac{24}{15}$ is the larger amount.

COMPARING THE SIZE OF A FRACTION TO A DECIMAL

Procedure

The table below shows you how to compare a fraction to a decimal in order to determine which number is larger.

Step	Action	Example
1	Identify the fraction and the decimal that you wish to compare.	Which is larger, .58 or $\frac{7}{12}$?
2	Convert the fraction to a decimal. Show the converted number one place more than the decimal to which it is being compared.	$\frac{7}{12}$ = .583 (.583 is one decimal place more than .58, to which it is being compared.)
3	Select the larger number.	.583 is larger than .58

Another example

Which is greater, .333 or $\frac{9}{32}$?

$\frac{9}{32}$ = .2812

.333 is greater than .2812.

PRACTICE

SOLUTIONS FOR COMPARING THE SIZES OF FRACTIONS BEGIN ON PAGE 45.

REINFORCEMENT PROBLEMS: COMPARING THE SIZES OF FRACTIONS

1. *Instructions:* Identify the *largest* number in the answer space provided. If the numbers are equivalent, write “equal” in the answer space.

Numbers	Answer	Numbers	Answer
a. $\frac{2}{3}, \frac{4}{9}$		f. $\frac{8}{7}, \frac{10}{9}$	
b. $\frac{11}{6}, \frac{4}{5}$		g. $\frac{3}{4}, \frac{9}{12}$	
c. $\frac{3}{7}, \frac{2}{5}$		h. $\frac{2}{5}, \frac{4}{5}$	
d. $\frac{11}{8}, \frac{17}{12}$		i. $1\frac{2}{4}, 1\frac{4}{5}$	
e. $\frac{8}{12}, \frac{3}{4}$		j. $7\frac{5}{8}, 4\frac{11}{14}$	

2. *Instructions:* In the table below, identify the larger number.

Numbers	Answer	Numbers	Answer
a. $\frac{8}{10}, .925$		e. $\frac{23}{19}, 1.25$	
b. $\frac{14}{17}, .85$		f. $\frac{9}{5}, 1.8$	
c. $\frac{3}{11}, .2727$		g. $\frac{2}{7}, .0295$	
d. $\frac{19}{15}, 1.25$		h. $\frac{99}{8}, 12.0375$	

---

3. *Instructions:* Calculate answers to the following questions:

   a. Andrea’s recipe for spaghetti requires 4/9 cup of red wine. Marlowe’s recipe for beef stew requires 3/7 cup of red wine. Which recipe requires the greatest amount of wine?

   b. The stock of Nguyen Company increased by $\$\frac{5}{8}$ today. The stock of Huynh Company increased by $\$\frac{9}{16}$ today. Which stock increased in value the most?

PRACTICE

3, *continued*

c. Knoxville Company incurred total expenses of $3,500 in the current year, of which $750 was for advertising. Nashville Company incurred total expenses of $1,400 of which $350 was for advertising. Which company paid the largest fractional portion for advertising expense?

SOLUTIONS

PRACTICE QUESTIONS FOR COMPARING THE SIZES OF FRACTIONS BEGIN ON PAGE 44.

REINFORCEMENT PROBLEMS: COMPARING THE SIZES OF FRACTIONS

1. a. $\frac{2}{3}$
   b. $\frac{11}{6}$
   c. $\frac{3}{7}$
   d. $\frac{17}{12}$
   e. $\frac{3}{4}$
   f. $\frac{8}{7}$
   g. equal
   h. $\frac{4}{5}$
   i. $1\frac{4}{5}$
   j. $7\frac{5}{8}$

2. a. .925
   b. .85
   c. $\frac{3}{11}$
   d. $\frac{19}{15}$
   e. 1.25
   f. equal
   g. $\frac{2}{7}$
   h. $\frac{99}{8}$

3. a. Andrea's equivalent fraction is 28/63. Marlowe's is 27/63. Andrea's recipe requires more.
   b. Nguyen Company's equivalent fraction is 80/128. Huynh Company's equivalent fraction is 72/128. Nguyen stock increased more (comparable decimals are .625 and .5625).
   c. Knoxville advertising can be expressed as the fraction 750/3500. Nashville advertising can be shown as 350/1400. Reducing the fractions to lowest terms to make them more manageable, we get 3/14 for Knoxville and 1/4 for Nashville. Converting these to equivalent fractions, Knoxville is 12/56 and Nashville is 14/56. Nashville incurs a greater fraction of its total expenses for advertising than does Knoxville (comparable decimals are .2143 and .25).

▼ *Multiplying Fractions*

OVERVIEW

General rule

The general rule for multiplying fractions is this: multiply numerators by numerators and multiply denominators by denominators. An overview of the procedure for multiplying any fractions can also be expressed by using letters as variables to represent individual numbers:

$$\frac{x}{y} \bullet \frac{a}{b} = \frac{xa}{yb}$$

Specific procedures

The material in this topic covers the following procedures:

- multiplying a fraction by a whole number
- multiplying a fraction by a fraction
- multiplying mixed numerals
- multiplying a fraction by a decimal

HOW TO MULTIPLY A FRACTION BY A WHOLE NUMBER

Procedure

The table below and on page 47 shows how to multiply a fraction by a whole number.

Step	Action	Example
1	Identify two numbers that you want to multiply.	Multiply 3 times $\frac{10}{12}$
2	Express the whole number as a fraction.	$\frac{3}{1}$
3	• Multiply numerators to get the numerator in the product. • Multiply denominators to get the denominator in the product. *Note:* You can also use cancellation as a shortcut (see page 48).	$\frac{3 \times 10}{1 \times 12} = \frac{30}{12}$

HOW TO MULTIPLY A FRACTION BY A WHOLE NUMBER (continued)

Procedure (continued)

Step	Action	Example
4	If the answer is not in lowest terms, reduce it to lowest terms by using the GCD method (see page 33).	$\frac{30}{12} = \frac{5}{2}$
5 *(optional)*	You can express a fraction greater than 1 as a mixed numeral.	$\frac{5}{2} = 2\frac{1}{2}$

Another example

1. Multiply 25 times $\frac{9}{39}$
2. The whole expressed as fraction is $\frac{25}{1}$
3. Multiplying: $\frac{25 \times 9}{1 \times 39} = \frac{225}{39}$
4. The fraction can be reduced to $\frac{75}{13}$

HOW TO MULTIPLY A FRACTION BY A FRACTION

Procedure

The table below shows you the procedure for multiplying two fractions.

Step	Action	Example
1	Identify two fractions that you want to multiply.	Multiply $\frac{2}{7}$ times $\frac{5}{4}$
2	• Multiply numerators to get the numerator in the product. • Multiply denominators to get the denominator in the product. *Note:* You can also use cancellation as a shortcut (see page 48).	$\frac{2 \times 5}{7 \times 4} = \frac{10}{28}$
3	If the answer is not in lowest terms, reduce it to lowest terms by using the GCD method (see page 33).	$\frac{5}{14}$
4 *(optional)*	You can express a fraction greater than 1 as a mixed numeral.	Fraction is less than 1

HOW TO MULTIPLY A FRACTION BY A FRACTION (continued)

Another example

- Multiply $\frac{75}{550}$ times $\frac{11}{12}$
- We multiply as follows: $\frac{75 \times 11}{550 \times 12} = \frac{825}{6600}$
- As usual, we reduce the answer, if possible. This results in $\frac{825}{6600}$ being reduced to $\frac{1}{8}$

Use cancellation to save time

Before multiplying fractions, you can use a quick procedure called cancellation that reduces the size of the fraction and makes multiplication easier. Cancellation also results in an answer that is smaller and often reduced to lowest terms.

Definition

Cancellation is the procedure of finding the same factor (a number that is multiplied) in both the numerator and denominator, and then eliminating that factor. (When a factor is eliminated, it is replaced by 1, although the 1 is usually not shown.)

Example #1

- Multiply $3 \times \frac{10}{12}$
- This results in multiplying: $\frac{3 \times 10}{1 \times 12}$
- Before multiplying, see if you can find the same numbers as factors in the both the numerator and the denominator: $\frac{3 \times \overbrace{\mathbf{5} \times \mathbf{2}}^{10}}{1 \times \underbrace{\mathbf{2} \times \mathbf{2} \times \mathbf{3}}_{12}}$

Result: You now have both 2s and 3s in the numerator and denominator.

- Now just cancel a number above for one of the same below: $\frac{\cancel{3} \times \mathbf{5} \times \cancel{\mathbf{2}}}{1 \times \mathbf{2} \times \cancel{\mathbf{2}} \times \cancel{\mathbf{3}}}$
- Multiply any remaining numbers in the numerator and denominator:

$$\frac{5}{1 \times 2} = \frac{5}{2}$$

HOW TO MULTIPLY A FRACTION BY A FRACTION (continued)

Example #2

- Multiply $\frac{75}{550}$ times $\frac{11}{12}$

Examine the numbers to find common factors. With a little trial and error, you can find:

- You can write the fraction as $\frac{\mathbf{3} \times \mathbf{25} \times 11}{\mathbf{22} \times \mathbf{25} \times 12}$, which can be further factored.

The 22 can be factored to $\frac{3 \times 25 \times 11}{\mathbf{2} \times \mathbf{11} \times 25 \times 12}$, which you can factor even more!

The 12 can also be factored: $\frac{3 \times 25 \times 11}{2 \times 11 \times 25 \times \mathbf{3} \times \mathbf{4}}$

- Now cancel out the same numbers above and below: $\frac{\cancel{3} \times \cancel{25} \times \cancel{11}}{2 \times \cancel{11} \times \cancel{25} \times \cancel{3} \times 4}$
- Multiply any remaining numbers: $\frac{1}{2 \times 4} = \frac{1}{8}$

Example #3

- Multiply $\frac{2}{7}$ times $\frac{5}{4}$
- Which is $\frac{2 \times 5}{7 \times 4}$, and can be factored to: $\frac{2 \times 5}{7 \times \mathbf{2} \times \mathbf{2}}$
- Cancelling, we get: $\frac{\cancel{2} \times 5}{7 \times \cancel{2} \times 2} = \frac{5}{14}$

HOW TO MULTIPLY MIXED NUMERALS

Procedure

If the multiplication involves mixed numerals:

- Convert the mixed numerals to improper fractions. (This procedure is discussed on page 23.)
- Then follow the same steps you learned in the table on page 47.

Example

Multiply: $3\frac{2}{7}$ times $\frac{1}{4}$

- Convert mixed numerals to improper fractions: $3\frac{2}{7} = \frac{23}{7}$
- Complete the multiplication: $\frac{23}{7} \times \frac{1}{4} = \frac{23}{28}$

HOW TO MULTIPLY A FRACTION BY A DECIMAL

Procedure

If you have to multiply a decimal and a fraction, it is generally most accurate to:

- Convert the decimal into a fraction (see page 38), and then
- Multiply the fractions.

Alternatively, you can also treat the decimal as if it were a whole number. This will result in an answer in decimal form, and may require rounding, which does not happen if you use only fractions. However, the alternative method is usually best when you are dealing with dollar amounts.

Example

Multiply 5.32 times $\frac{3}{8}$.

Converting the decimal to a fraction, and multiplying:

$$5\frac{32}{100}\times\frac{3}{8}=\frac{532}{100}\times\frac{3}{8}=\frac{1596}{800}$$

Reducing the fraction to lowest terms, we get: $\frac{399}{200}$

Alternatively:

$$\frac{5.32}{1}\times\frac{3}{8}=\frac{5.32\times 3}{1\times 8}=\frac{15.96}{8}=1.995$$

Last step: divide by denominator

HOW TO MULTIPLY MORE THAN TWO FRACTIONS

Procedure

If you need to multiply a series a fractions, just multiply them in sequence, two at a time, following all the same procedures you learned above. (You can also still do the same cancellation procedures, if you desire.)

Example

Multiply: $\frac{3}{14}$, $2\frac{2}{3}$ and $\frac{2}{5}$.

- First, $\frac{3}{14}\times\frac{8}{3}=\frac{3\times 8}{14\times 3}=\frac{24}{42}=\frac{4}{7}$ *Note:* $2\frac{2}{3}$ became $\frac{8}{3}$.
- Next, $\frac{4}{7}\times\frac{2}{5}=\frac{4\times 2}{7\times 5}=\frac{8}{35}$, which cannot be reduced.

HOW TO MULTIPLY MORE THAN TWO FRACTIONS (continued)

Language used to indicate multiplying fractions

To indicate that you need to multiply with fractions, different phrases may be used. Usually these phrases include the name of the fraction and the word "of." These highlighted expressions below indicate multiplication that involves fractions:

- "**One-third of** $400 is what amount?"
- "Take **five-sevenths of** that amount."
- "**Three-eighths** is what **portion of** two-thirds?"
- "What **fraction of** the total is still available?"

Example

"Blackstone Company has spent **three-eighths of** its total budget on wages expense. If the total budget is $524,000, what amount was spent on wages?"

Answer: $\frac{3}{8} \times \$524{,}000 = \frac{3}{8} \times \frac{524{,}000}{1} = \frac{1{,}572{,}000}{8} = \$196{,}500$

PRACTICE

SOLUTIONS FOR MULTIPLYING FRACTIONS BEGIN ON PAGE 54.

REINFORCEMENT PROBLEMS: MULTIPLYING FRACTIONS

1. *Instructions:* Multiply and show all fractional answers in lowest terms. Write your answer in the space provided next to each item.

Item	Answer	Item	Answer	Item	Answer
a. $3 \times \frac{2}{5}$		h. $\frac{40}{100} \times \frac{12}{8}$		o. $\frac{10}{11} \times \frac{11}{5}$	
b. $\frac{2}{10} \times \frac{4}{6}$		i. $\frac{37}{85} \times 4\frac{2}{3}$		p. $750 \times \frac{1}{6}$	
c. $5\frac{2}{3} \times \frac{3}{8}$		j. $\frac{1}{8} \times \frac{3}{4}$		q. $\frac{9}{10} \times \frac{3}{4}$	
d. $1{,}500 \times \frac{11}{15}$		k. $\frac{1}{8} \times \frac{3}{4} \times \frac{4}{5}$		r. $\frac{15}{25} \times \frac{14}{30} \times \frac{13}{20}$	
e. $\frac{6}{8} \times \frac{2000}{50}$		l. $200 \times \frac{3}{4}$		s. $\frac{a}{b} \times \frac{x}{y}.$	
f. $\frac{9}{17} \times \frac{12}{27}$		m. $12\frac{4}{5} \times 7\frac{6}{7}$		t. $5 \times \frac{x}{y}$	
g. $\frac{2}{5} \times \frac{3}{4}$		n. $\frac{5}{6} \times \frac{3}{8}$		u. $\frac{1}{b} \times \frac{x}{y}$	

2. Perform the following calculations that involve decimals. Round to the thousandths place, if necessary, except when using dollars. Round to the hundredths place for dollar amounts. Fractional answers should be in lowest terms.

Item	Answer	Item	Answer	Item	Answer
a. $\frac{1}{3} \times .4$		d. $5{,}250 \times \frac{15}{12}$		g. $.25 \times \frac{a}{b}$	
b. $\$20.75 \times \frac{5}{9}$		e. $9.582 \times 2\frac{1}{8}$		h. $.25 \times a\frac{b}{c}$	
c. $.005 \times 1\frac{4}{5}$		f. $\frac{7}{8} \times 1.75$		i. $\$250.30 \times \frac{2}{5}$	

PRACTICE

SOLUTIONS FOR MULTIPLYING FRACTIONS BEGIN ON PAGE 54.

3. Calculate answers to the following questions.

   a. If Dave walks $3\frac{1}{8}$ miles per hour, how far does he walk in $5\frac{1}{2}$ hours?

   b. You are doing adjusting journal entries at the end of the year. You discover that one-fourth of the prepaid insurance has been used up, but this has not been recorded yet. The amount of the prepaid insurance showing in the ledger is $900. What is the amount of the adjustment?

   c. Ace Company has a payroll tax expense of $1,225.50 for each five-day week, which ends on Fridays. However, the current accounting period ends on a Thursday. What is the amount of the payroll tax expense that Ace Company needs to record?

   d. Zydek Company has a total expense budget of $180,000. One-eighth of this budget is allocated to utilities expense. So far, five-twelfths of the utilities budget has been spent.

      (1) What dollar amount has been allocated to utilities?

      (2) What fractional portion of the total budget has been spent on utilities so far?

   e. The Able, Baker, and Cooper partnership shares profits and losses in the ratio of one-eighth to Able, three-eighths to Baker, and four-eighths to Cooper. However, Zweig, a new partner, now enters the partnership and will be allocated a $\frac{2}{5}$ share of all profits and losses.

      (1) What fractional share of total partnership profits and losses will the old partners as a group be allocated?

      (2) What fractional share will each of the old partners now receive individually?

SOLUTIONS

PRACTICE QUESTIONS FOR MULTIPLYING FRACTIONS BEGIN ON PAGE 52.

REINFORCEMENT PROBLEMS: MULTIPLYING FRACTIONS

1.

Item	Answer	Item	Answer	Item	Answer
a. $3 \times \frac{2}{5}$	6/5	h. $\frac{40}{100} \times \frac{12}{8}$	3/5	o. $\frac{10}{11} \times \frac{11}{5}$	2
b. $\frac{2}{10} \times \frac{4}{6}$	2/15	i. $\frac{37}{85} \times 4\frac{2}{3}$	74/255	p. $750 \times \frac{1}{6}$	125
c. $5\frac{2}{3} \times \frac{3}{8}$	17/8 (or 2 1/8)	j. $\frac{1}{8} \times \frac{3}{4}$	3/32	q. $\frac{9}{10} \times \frac{3}{4}$	27/40
d. $1{,}500 \times \frac{11}{15}$	1,100	k. $\frac{1}{8} \times \frac{3}{4} \times \frac{4}{5}$	3/40	r. $\frac{15}{25} \times \frac{14}{30} \times \frac{13}{20}$	91/500
e. $\frac{6}{8} \times \frac{2000}{50}$	30	l. $200 \times \frac{3}{4}$	150	s. $\frac{a}{b} \times \frac{x}{y}$	*ax/by*
f. $\frac{9}{17} \times \frac{12}{27}$	4/17	m. $12\frac{4}{5} \times 7\frac{6}{7}$	704/7 (or 100 4/7)	t. $5 \times \frac{x}{y}$	*5x/y*
g. $\frac{2}{5} \times \frac{3}{4}$	3/10	n. $\frac{5}{6} \times \frac{3}{8}$	5/16	u. $\frac{1}{b} \times \frac{x}{y}$	*x/by*

2.

Item	Answer
a. $\frac{1}{3} \times .4$	$\frac{1}{3} \times \frac{4}{10} = \frac{4}{30} = \frac{2}{15}$ Alternatively: $\frac{1}{3} \times \frac{.4}{1} = \frac{.4}{3} = .133$
b. $\$20.75 \times \frac{5}{9}$	$\$20\frac{75}{100} \times \frac{5}{9} = \frac{2075}{100} \times \frac{5}{9} = \$\frac{10{,}375}{900} = \$\frac{415}{36} = \$11\frac{19}{36}$ Alternatively: $\$\frac{20.75}{1} \times \frac{5}{9} = \frac{103.75}{9} = \$11.53$ (Rounded to nearest cent. When using $, this method gives a more understandable answer, doesn't it?)
c. $.005 \times 1\frac{4}{5}$	$\frac{5}{1000} \times \frac{9}{5} = \frac{45}{5000} = \frac{9}{1{,}000}$ Alternatively: $\frac{.005}{1} \times \frac{9}{5} = \frac{.045}{5} = .009$
d. $5{,}250 \times \frac{15}{12}$	This is a whole number, so: $\frac{5{,}250}{1} \times \frac{15}{12} = \frac{78{,}750}{12} = 6{,}562\frac{1}{2}$ or 6,526.5
e. $9.582 \times 2\frac{1}{8}$	$9\frac{582}{1{,}000} \times \frac{17}{8} = \frac{9{,}582}{1{,}000} \times \frac{17}{8} = \frac{162{,}894}{8{,}000} = \frac{81{,}447}{4{,}000}$ Alternatively: $\frac{9.582}{1} \times \frac{17}{8} = \frac{162.894}{8} = 20.362$ (rounded)
f. $\frac{7}{8} \times 1.75$	$\frac{7}{8} \times 1\frac{75}{100} = \frac{7}{8} \times \frac{175}{100} = \frac{1{,}225}{800} = 1\frac{17}{32}$ Alternatively: $\frac{7}{8} \times \frac{1.75}{1} = \frac{12.25}{8} = 1.531$ (rounded)
g. $.25 \times \frac{a}{b}$	$\frac{25}{100} \times \frac{a}{b} = \frac{25a}{100b} = \frac{a}{4b}$

SOLUTIONS

PRACTICE QUESTIONS FOR MULTIPLYING FRACTIONS BEGIN ON PAGE 52.

2, *continued*

Item	Answer
h. $.25 \times a\frac{b}{c}$	$\frac{25}{100} \times a\frac{b}{c} = \frac{25}{100} \times \frac{(ac+b)}{c} = \frac{25(ac+b)}{100c} = \frac{(ac+b)}{4c}$
i. $\$250.30 \times \frac{2}{5}$	$\$250\frac{30}{100} \times \frac{2}{5} = \frac{25,030}{100} \times \frac{2}{5} = \$\frac{50,060}{500} = \$\frac{2,503}{25}$ Alternatively: $\$\frac{250.30}{1} \times \frac{2}{5} = \frac{500.60}{5} = \$100.12$

3. a. $3\frac{1}{8} \times 5\frac{1}{2} = \frac{25}{8} \times \frac{11}{2} = \frac{275}{16} = 17\frac{3}{16}$ miles

b. $\frac{1}{4} \times \$900 = \frac{1}{4} \times \frac{900}{1} = \frac{900}{4} = \$225$

c. $\frac{4}{5} \times \$1,225.50 = \frac{4}{5} \times \frac{1,225.5}{1} = \frac{4,902}{5} = \$980.40$

d. (1) The dollar amount allocated to the utilities is $\$180,000 \times \frac{1}{8} = \$22,500$

(2) The fractional portion of the total budget spent on utilities so far is $\frac{1}{8} \times \frac{5}{12} = \frac{5}{96}$

e. (1) Because Zweig, the new partner, is receiving $\frac{2}{5}$, that leaves $\frac{3}{5}$ for all the old partners as a group.

(2) Able receives $\frac{1}{8}$ of the $\frac{3}{5}$, which is $\frac{3}{8} \times \frac{3}{5} = \frac{9}{40}$ share. Baker receives $\frac{3}{8}$ of the $\frac{3}{5}$, which is $\frac{3}{8} \times \frac{3}{5} = \frac{9}{40}$ share. Cooper receives $\frac{4}{8}$ of the $\frac{3}{5}$, which is $\frac{4}{8} \times \frac{3}{5} = \frac{12}{40}$ share.

Note: It would probably be easiest if the partnership started measuring all shares in units of fortieths, with the new partner converting his two-fifths share into an equivalent fraction of sixteen-fortieths.

Dividing Fractions

RECIPROCALS

Overview

The basic idea for division as applied to fractions is the same as for any other numbers. However, for fractions there is a procedure that simplifies the operation. This procedure is called using a **reciprocal**.

Example: why it is done

Suppose that you want to divide the fraction $\frac{10}{12}$ by 2. Because 2 expressed as a fraction is $\frac{2}{1}$, we would certainly obtain the correct answer if we divided the numerator by 2 and divided the denominator by 1, like this: $\frac{10 \div 2}{12 \div 1} = \frac{5}{12}$. The answer makes sense. 5 parts out of 12 is half as much as 10 parts out of 12. However, even though the above approach does work, it can create some unnecessarily awkward and difficult answers. Suppose that you want to divide the same amount, $\frac{10}{12}$, by 7. If you do this: $\frac{10 \div 7}{12 \div 1}$, you get $\frac{1\frac{3}{7}}{12}$ which is a pretty ugly-looking number, and awkward to deal with. And it can get even worse than that! Fortunately, there is a better alternative. There is a much easier way that gives nice, clean fractions.

A reciprocal

To show the reciprocal of any number, invert the number.

Example: The reciprocal of 7 is $\frac{1}{7}$

Other examples

The reciprocal of 3 is $\frac{1}{3}$

The reciprocal of $\frac{1}{3}$ is 3

The reciprocal of $\frac{4}{3}$ is $\frac{3}{4}$

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

HOW TO DIVIDE FRACTIONS

Rule for dividing fractions

To divide a fraction, **multiply** the dividend by the **reciprocal** of the divisor.

A generalization of this procedure for dividing fractions can also be expressed by using letters as variables to represent individual numbers:

$$\frac{x}{y} \div \frac{a}{b} = \frac{x}{y} \times \frac{b}{a} = \frac{xb}{ya}$$

Example

To divide $\frac{10}{12}$ by 7, do this: $\frac{10}{12} \times \frac{1}{7} = \frac{10}{84}$ which is also reducible to $\frac{5}{42}$

Procedure

The table below gives you a more detailed procedure for division that involves fractions.

Step	Action	Example
1	Identify the dividend and the divisor. *Note:* A divisor is the number after the ÷ symbol, or the word "by," or before the word "into."	Divide $\underset{\underset{\text{dividend}}{\nearrow}}{2\frac{1}{4}}$ by $\underset{\underset{\text{divisor}}{\uparrow}}{\frac{3}{5}}$ Therefore, we are saying: $2\frac{1}{4} \div \frac{3}{5}$
2	Convert mixed numerals or whole numbers into fractions.	$2\frac{1}{4}$ becomes $\frac{9}{4}$
3	- Change the division sign to a multiplication sign. - Replace the divisor with its reciprocal.	$\frac{9}{4} \times \frac{5}{3}$
4	Follow the usual procedure for multiplying fractions, including cancellation.	$\frac{3\cdot\cancel{3}}{4} \times \frac{5}{\cancel{3}} = \frac{15}{4} = 3\frac{3}{4}$

PRACTICE

SOLUTIONS FOR DIVIDING FRACTIONS BEGIN ON PAGE 60.

REINFORCEMENT PROBLEMS: DIVIDING FRACTIONS

1. *Instructions:* Divide and show all fractional answers in lowest terms. Write your answer in the space provided next to each item.

Item	Answer	Item	Answer	Item	Answer
a. $\frac{1}{3} \div \frac{2}{5}$		g. $4\frac{4}{5} \div 3\frac{3}{9}$		m. $\frac{1}{12} \div \frac{1}{12}$	
b. $\frac{9}{7} \div \frac{3}{8}$		h. $\frac{36}{85} \div \frac{44}{99}$		n. $750 \div \frac{1}{5}$	
c. $5\frac{1}{8} \div \frac{1}{12}$		i. $\frac{1}{8} \div \frac{3}{4}$		o. $\frac{9}{10} \div \frac{1}{2x}$	
d. $22\frac{9}{17} \div 7\frac{2}{3}$		j. $\frac{1}{8} \div \frac{3}{4} \div \frac{4}{5}$		p. $\frac{5}{10} \div \frac{x}{y}$	
e. $\frac{9}{3} \div \frac{12}{27}$		k. $12\frac{4}{5} \div 7\frac{6}{7}$		q. $5 \div \frac{x}{y}$	
f. $\frac{2}{5} \div \frac{3}{4}$		l. $\frac{5}{6} \div \frac{3}{8}$		r. $\frac{1}{b} \div \frac{x}{y}$	

---

2. Calculate the correct answer to each of the following problems.

   a. Mega Company reported that $\frac{8}{15}$ of its total sales revenue came from the Western sales region. This was $2\frac{2}{7}$ times the amount of sales reported from the Eastern region. What fractional portion of its total sales did Mega report from the Eastern region?

   b. A section of trench, which is to contain drain pipe, is $22\frac{7}{12}$ feet long. Each full available pipe section is $1\frac{4}{5}$ feet long. How many full sections of pipe will have to be purchased to have just enough pipe to complete the drain?

   c. A box containing books weighs 175 pounds. If each book weighs $3\frac{4}{7}$ pounds, how many books are in the box, assuming the weight of the box is minimal?

PRACTICE

SOLUTIONS FOR DIVIDING FRACTIONS BEGIN ON PAGE 60.

2, *continued*

d. Angela drove her car $245\frac{4}{10}$ miles, using a full tank of gas. When she stopped at a gas station, she refilled the tank with $12\frac{1}{8}$ gallons of gasoline. What was the average miles per gallon at which the car operated?

e. The owner of a business tells you that the remaining amount which should be showing in the Prepaid Rent account is $2,100. This is $\frac{7}{12}$ of the amount that was originally prepaid five months ago. What was the amount that was prepaid five months ago?

f. The price/earnings ratio of a stock shows what multiple the price per share of a stock is, compared to the net income per share. Generally, a multiple greater than 25 times indicates that a stock may be overpriced. If the stock price of Goofy Technology Corporation is $\$57\frac{1}{4}$ per share, and the net income is $1.30 per share, would you buy the stock? Express your answer as a fraction.

---

3. **Reducing to lowest terms and converting to mixed numerals.** One of the operations below is reducing a fraction to lowest terms. The other operation is converting a fraction to a mixed numeral. Which is which? What is the difference between the two operations?

a. $\frac{12}{8} = 1\frac{1}{2}$

b. $\frac{12}{8} = \frac{12 \div 4}{8 \div 4} = \frac{3}{2}$

---

4. **Decimal equivalents.** What is the decimal equivalent of the two examples below (round answers to 3 places, if necessary)?

a. $3\frac{4}{19}$

b. $\frac{8}{12}$

SOLUTIONS

PRACTICE QUESTIONS FOR DIVIDING FRACTIONS BEGIN ON PAGE 58.

REINFORCEMENT PROBLEMS: DIVIDING FRACTIONS

1.

Item	Answer	Item	Answer	Item	Answer
a. $\frac{1}{3} \div \frac{2}{5}$	5/6	g. $4\frac{4}{5} \div 3\frac{3}{9}$	1 11/25	m. $\frac{1}{12} \div \frac{1}{12}$	1
b. $\frac{9}{7} \div \frac{3}{8}$	24/7	h. $\frac{36}{85} \div \frac{44}{99}$	81/85	n. $750 \div \frac{1}{5}$	3,750
c. $5\frac{1}{8} \div \frac{1}{12}$	61 1/2	i. $\frac{1}{8} \div \frac{3}{4}$	1/6	o. $\frac{9}{10} \div \frac{1}{2x}$	$9x/5$
d. $22\frac{9}{17} \div 7\frac{2}{3}$	1149/391 (or 2 9/151)	j. $\frac{1}{8} \div \frac{3}{4} \div \frac{4}{5}$	5/24	p. $\frac{5}{10} \div \frac{x}{y}$	$y/2x$
e. $\frac{9}{3} \div \frac{12}{27}$	27/4	k. $12\frac{4}{5} \div 7\frac{6}{7}$	448/275 (or 1 173/275)	q. $5 \div \frac{x}{y}$	$5y/x$
f. $\frac{2}{5} \div \frac{3}{4}$	8/15	l. $\frac{5}{6} \div \frac{3}{8}$	20/9 (or 2 2/9)	r. $\frac{1}{b} \div \frac{x}{y}$	y/bx

2. a. $\frac{8}{15} \div 2\frac{2}{7} = \frac{7}{30}$ (In effect, you are asking: $\frac{8}{15}$ is $2\frac{2}{7}$ times (a multiple of) what amount?)

   b. $22\frac{7}{12} \div 1\frac{4}{5} = \frac{1355}{108}$ This needs to be converted to a mixed number so we can see how many pipe sections.

   $\frac{1355}{108} = 12\frac{59}{108}$ This is about 12 1/2 sections. So we will need to buy a full 13 sections of pipe in order to complete the job.

   c. $175 \div 3\frac{4}{7} = 49$ books are in the box.

   d. $245\frac{4}{10} \div 12\frac{1}{8} = \frac{9816}{485} = 20\frac{116}{485}$ miles per gallon (as a decimal, this is approximately 20.239 miles per gallon).

   e. $\$2,100 \div \frac{7}{12} = \$2,100 \times \frac{12}{7} = \$3,600$ Check: $2,100 is 7/12 of $3,600.)

   f. The price/earnings multiple is $\$57\frac{1}{4} \div \$1\frac{3}{10}$, which results in a multiple of $44\frac{1}{26}$, so this is a very expensive stock, unless the company is about ready to do fantastic things and earn a lot more money.

3. a. is converting the fraction to a mixed numeral, and b. is reducing the fraction to lowest terms. Converting to a mixed numeral is done by dividing the numerator by the denominator. A mixed numeral is simply an optional way of expressing *a fraction that is greater than 1*. Reducing a fraction to lowest terms is done by dividing the numerator and denominator by the same number so the resulting numbers are evenly divisible only by 1. This is done in order to make the fraction easier to read and use in calculations. In both situations A and B, *each result is equivalent in value to the original fraction*.
4. Decimal equivalents are: 3.211 and .667 (rounded).

Adding Fractions

GENERAL RULE FOR ADDING FRACTIONS

Rule

To add fractions, make sure the denominators are the same numeral. Then add the numerators, and write a new fraction with the total of the numerators over the same numeral for the denominator.

A generalization of this procedure for adding fractions can also be expressed by using letters as variables to represent individual numbers:

$$\frac{a}{y}+\frac{x}{y}=\frac{(a+x)}{y}$$

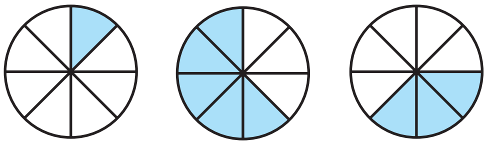
ADDING FRACTIONS WITH THE SAME DENOMINATOR

Overview

As you know, the denominator of a fraction shows the number of parts in a single whole amount of something. In this discussion, you will see how to add fractions that come from whole amounts that are made up of the same number of parts.

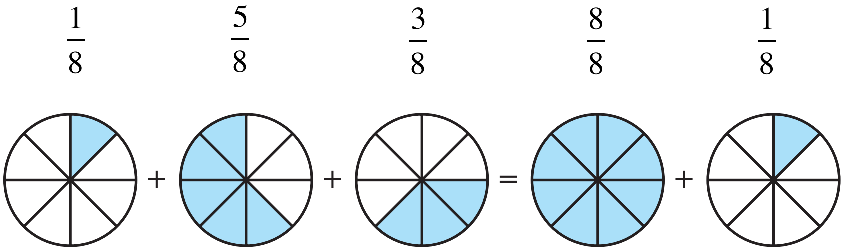
Example

Suppose that you want to add these fractions: $\frac{1}{8}$, $\frac{5}{8}$, and $\frac{3}{8}$. All of these fractions are parts of whole amounts that consist of eight parts. You could visualize the fractional parts of the whole amounts like this, where each figure is one whole amount with eight equal parts:



ADDING FRACTIONS WITH THE SAME DENOMINATOR (continued)

To graphically show the addition of these parts, we could show this:



So, $\frac{1}{8}+\frac{5}{8}+\frac{3}{8}=\frac{9}{8}$, which is slightly more than one full whole amount. You could also rewrite this improper fraction as a mixed numeral, which would be $1\frac{1}{8}$.

Procedure

The following table shows the procedure for adding fractions *that have the same denominator value*.

Step	Action	Example
1	Add the numerators.	Add the fractions $\frac{2}{10}, \frac{4}{10}$ therefore, $2 + 4 = 6$.
2	Show the answer as a new fraction with: - the same denominator, and - the numerator is the total from **STEP 1**.	$\frac{2}{10}+\frac{4}{10}=\frac{6}{10}$
3	If necessary, reduce the fraction to lowest terms.	$\frac{\cancel{2}\cdot 3}{\cancel{2}\cdot 5}=\frac{3}{5}$

ADDING MIXED NUMERALS WITH THE SAME DENOMINATOR

Procedure

The following table shows the procedure for adding mixed numerals where the fractional parts all *have the same denominator value*.

Step	Action	Example
1	Vertically align the expressions.	Add $2\frac{3}{7}+4\frac{5}{7}$: $2\frac{3}{7}$ $4\frac{5}{7}$
2	Add the whole numbers.	$2\frac{3}{7}$ $4\frac{5}{7}$ $\longrightarrow$ 6
3	- Add the fractions in the usual way for fractions with the same denominator. - Write the new fraction next to the whole number.	$2\frac{3}{7}$ $4\frac{5}{7}$ $6\frac{8}{7}$
4	If the fraction part of the answer is an improper fraction: - convert the fraction to a mixed numeral, then - add the mixed numeral to the whole number in the answer.	$\frac{8}{7}=1\frac{1}{7}$ then: $1\frac{1}{7}+6=7\frac{1}{7}$
5	Reduce the fraction to lowest terms, if necessary.	Not necessary here.

PRACTICE

REINFORCEMENT PROBLEMS: ADDING FRACTIONS WITH THE SAME DENOMINATOR

Instructions: Add and show all fractional answers in lowest terms. Adjust answers that are mixed numerals so that the fractional part is never equal to or greater than 1. Write your answer in the space provided next to each item.

Item	Answer	Item	Answer	Item	Answer
1. $\frac{4}{11}+\frac{3}{11}$		5. $5+12\frac{7}{8}$		9. $15\frac{10}{12}+3\frac{8}{12}$	
2. $4\frac{2}{3}+\frac{2}{3}$		6. $\frac{1}{4}+4\frac{3}{4}+\frac{5}{4}$		10. $3+5+\frac{1}{9}$	
3. $2\frac{18}{21}+28\frac{12}{21}$		7. $2\frac{7}{9}+5\frac{5}{9}$		11. $3\frac{4}{5}+\frac{3}{5}$	
4. $\frac{1}{5}+\frac{3}{5}$		8. $\frac{5}{17}+\frac{12}{17}$		12. $\frac{a}{x}+\frac{b}{x}$	

SOLUTIONS

Item	Answer	Item	Answer	Item	Answer
1. $\frac{4}{11}+\frac{3}{11}$	7/11	5. $5+12\frac{7}{8}$	17 7/8	9. $15\frac{10}{12}+3\frac{8}{12}$	19 1/2
2. $4\frac{2}{3}+\frac{2}{3}$	5 1/3	6. $\frac{1}{4}+4\frac{3}{4}+\frac{5}{4}$	6 1/4	10. $3+5+\frac{1}{9}$	8 1/9
3. $2\frac{18}{21}+28\frac{12}{21}$	31 3/7	7. $2\frac{7}{9}+5\frac{5}{9}$	8 1/3	11. $3\frac{4}{5}+\frac{3}{5}$	4 2/5
4. $\frac{1}{5}+\frac{3}{5}$	4/5	8. $\frac{5}{17}+\frac{12}{17}$	1	12. $\frac{a}{x}+\frac{b}{x}$	$(a + b)/x$

ADDING MIXED NUMERALS WITH THE SAME DENOMINATOR

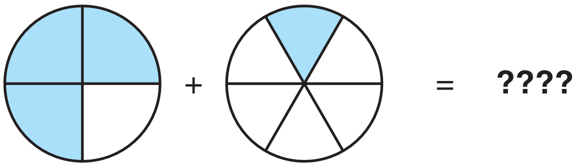
Overview

You have probably noticed that so far when we added fractions, we only added fractions that have the same denominators. What if we need to add fractions that have different denominators? We convert the fractions into equivalent fractions that have the same denominators. Then we are able to add.

Example

Suppose that we have two fractions: $\frac{3}{4}$ and $\frac{1}{6}$

If we wish to add these fractional values, we are saying that we want to add three pieces from a whole amount that has four parts to one piece from a whole amount that has six parts.



But the parts are different sizes! This is like trying to add three slices of an apple to one slice of an orange. We cannot get one total that is made up of either type of slice because we are not adding the same things. Or, in the above example, we cannot get an answer in either fourths or sixths, because the fourths and sixths are pieces of different sizes—they are different things.

Solution

To solve this problem, we convert each of the fractions into equivalent-value fractions that have the same denominators, so all the parts will be the same size.

WHAT IS THE LOWEST COMMON DENOMINATOR?

Find equivalent fractions

To find the correct equivalent fractions, we look for fractions that are multiples of the fractions we want to add. The fractions that we find must also have the same denominator that is a common multiple of the denominators in the fractions we are adding.

WHAT IS THE LOWEST COMMON DENOMINATOR? (continued)

Example of common multiples

To find a common multiple of two numbers, first multiply each number by 1, 2, 3, etc. Suppose we want to find the common multiples of 4 and 6:

- Multiples of 4 are: 4, 8, 12, 16, 20, 24, 28, 32, 36, and so on …
- Multiples of 6 are: 6, 12, 18, 24, 30, 36, 42, 48, and so on …

If you look at the numbers above, you can see that common multiples of *both* 4 and 6 are **12**, **24**, and **36**. We could use any of these multiples as a common multiple for our fractions, but it is always easiest to work with the smallest multiple possible. The smallest common multiple is called the **lowest common multiple**. In this example, the lowest common multiple is 12.

Lowest common denominator

Whenever a lowest common multiple is used for the purpose of converting fractions with unlike denominators to equivalent fractions with the same denominator, that multiple is called the **lowest common denominator**. Here we will call it the “**LCD**.”

HOW TO ADD FRACTIONS USING THE LCD

Procedure

The table on page 67 shows how to add unlike fractions, using the examples of $\frac{3}{4}$ and $\frac{1}{6}$.

Before you begin

Before you begin, you must identify the lowest common denominator of the fractions that you wish to add. In this example, we have already identified it as 12 (see above)

HOW TO ADD FRACTIONS USING THE LCD (continued)

Step	Action	Example
1	- Select the first fraction you are adding. - Next to it, write a fraction bar with the LCD as denominator.	$\frac{3}{4} \Rightarrow \frac{\quad}{12}$
2	Change the unfinished fraction into a fraction that is equivalent to what you are adding: - Divide the LCD by the old denominator to *determine the multiple* that results from using the LCD. - Use this multiple to multiply times the old numerator to obtain the new numerator. - Write the new equivalent fraction.	- $\underbrace{\frac{3}{4} \Rightarrow \frac{\quad}{12}}_{12 \div 4 = 3}$ (multiple is 3) - $\overbrace{\frac{3}{4} \Rightarrow \frac{9}{12}}^{3 \times 3 = 9}$ - Result: $\frac{9}{12}$ $\left(\text{which is really } \frac{3 \times \mathbf{3}}{4 \times \mathbf{3}}\right)$
3	Repeat **STEPS 1** and **2** for the next fraction to be added.	- $\frac{1}{6} \Rightarrow \frac{\quad}{12}$ - $\underbrace{\frac{1}{6} \Rightarrow \frac{\quad}{12}}_{12 \div 6 = 2}$ - $\overbrace{\frac{1}{6} \Rightarrow \frac{2}{12}}^{1 \times 2 = 2}$
4	Add the new fractions in the usual way for fractions with the same denominator.	$\frac{9}{12} + \frac{2}{12} = \frac{11}{12}$

Caution!

Common mistake!

Cancellation *does not work* when adding fractions.

When adding fractions, *cancellation will not give the correct answer*.

$$\frac{2}{5} + \frac{5}{6} = \frac{\cancel{2} \cdot 1}{\cancel{5} \cdot 1} + \frac{5 \cdot 1}{3 \cdot \cancel{2}} \quad \longleftarrow \boxed{\textbf{NO!}}$$

PRACTICE

REINFORCEMENT PROBLEMS: ADDING FRACTIONS

Instructions: Use the procedure presented above to add fractions that have different denominators. First, find the LCD and then follow the procedure. Write your answer in the space provided.

Add these …	LCD is …	Answer
1. $\frac{2}{3}+\frac{3}{4}$		
2. $\frac{1}{2}+\frac{3}{5}$		
3. $\frac{4}{7}+\frac{5}{14}$		
4. $\frac{7}{5}+\frac{3}{4}$		
5. $\frac{5}{8}+\frac{3}{12}$		

SOLUTIONS

Add these …	LCD is …	Answer
1. $\frac{2}{3}+\frac{3}{4}$	12	1 5/12
2. $\frac{1}{2}+\frac{3}{5}$	10	1 1/10
3. $\frac{4}{7}+\frac{5}{14}$	14	13/14
4. $\frac{7}{5}+\frac{3}{4}$	20	2 3/20
5. $\frac{5}{8}+\frac{3}{12}$	24	7/8

▼ *Three Ways to Find the Lowest Common Denominator*

OVERVIEW

Introduction

So far, the unlike fractions with which you have practiced have had lowest common denominators that were relatively easy to determine. This is not always the case in practice. If you are to become competent in adding factions with different denominators, you will need to know more about how to find the LCD.

Three methods

There are three methods you can use to find the LCD. Of course, you always try to use the easiest method whenever possible. However, the complexity of the fractions that you are adding usually dictates which method you need to use.

METHOD #1

Multiplying denominators individually

Multiply the denominator of each fraction by 1, 2, 3, etc., until you find the lowest common multiple. This is what we have done so far. This method works relatively well for smaller denominators—generally between 1 and 10.

Example

You want to add: $\frac{1}{4} + \frac{9}{10}$. Find the multiples of each denominator by multiplying each one times 1, 2, 3, etc., until you see the first common multiple.

- Multiples of 4: 4, 8, 12, 16, **20**
- Multiples of 10: 10, **20**

So, 20 is the lowest common denominator.

METHOD #2

Multiplying denominators by each other

Multiply the denominators by each other. This will always give you a common denominator, but *it may not be the lowest common denominator*. Multiplying like this is perfectly acceptable if the numbers are small—say, less than 10. Above this, the common denominator becomes too big, and makes completing the procedure awkward because of the size of the numbers.

METHOD #2 (continued)

Example

To find a common denominator for $\frac{1}{4} + \frac{9}{10}$, you could simply multiply 4 times 10 to get 40. 40 is not the lowest common denominator, but it is workable. You can reduce the answer to lowest terms later.

METHOD #3

Prime number factoring

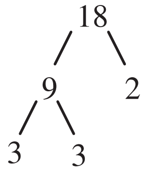
This method is called the **prime number factoring method**. This is the method you use for big, difficult numbers and when the other two methods do not work easily. The lowest prime number factoring method will always find the lowest common denominator in every situation, so it is good to know.

Factoring

A factor is a number that is being multiplied. Factoring is an amusing way to see what factors a number can be broken into. For example, try the number 18. This number can be factored into the factors of 9 × 2.

Lowest factors

But can we find more factors of 18? Yes, because 9 can be further factored into 3 × 3. So it turns out that the lowest factors that 18 can be factored into are 3 × 3 × 2!



It does not matter what numbers you begin with. For example, you could have initially factored 18 into 6 × 3. You would still end up with the final factors of 3 × 3 × 2.

Prime numbers

A prime number is a whole number larger than 1 that is evenly divisible only by 1 and itself. Here are the first 20 prime numbers:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, and 71

An amazing thing about prime numbers is that any whole number above 1 that is not a prime number can be factored into a product of prime numbers! This is what happened with the number 18 that you see above.

METHOD #3 (continued)

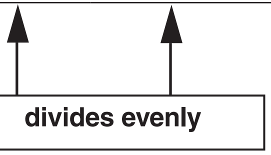
Primes and the LCD

The lowest common denominator (LCD) of any group of fractions will always be the product of prime numbers of the denominators. The following procedure table shows you how to find the prime numbers to determine the LCD.

How to find the LCD using prime factoring

The table below shows the steps for a technique that finds the LCD by factoring prime numbers. Here, we want to add the fractions $\frac{9}{10}$ and $\frac{7}{12}$.

<table>
<tr><th>Step</th><th>Action</th></tr>
<tr><td>1</td><td>Write the denominators of the fractions next to each other in a box.
$\boxed{10 \quad 12}$</td></tr>
<tr><td>2</td><td>Select the lowest prime number (2) that divides evenly into a denominator, and write that number to the left of the box.
$2\ \boxed{10 \quad 12}$</td></tr>
<tr><td>3</td><td>Divide the prime number into the numbers in the box, which are now dividends, for purposes of division.
<table>
<tr><th>IF …</th><th>THEN …</th></tr>
<tr><td>the divisor (the prime number) divides evenly into the dividend,</td><td>bring down the <i>quotient</i> and write it directly under the dividend below the box.</td></tr>
<tr><td>the divisor does <i>not</i> divide evenly into the dividend</td><td>bring down the <i>dividend</i> and write it directly under the same number below the box.</td></tr>
</table>
$\begin{array}{c|cc|} \cline{2-3} 2 & 10 & 12 \\ \cline{2-3} & 5 & 6 \end{array}$</td></tr>
</table>

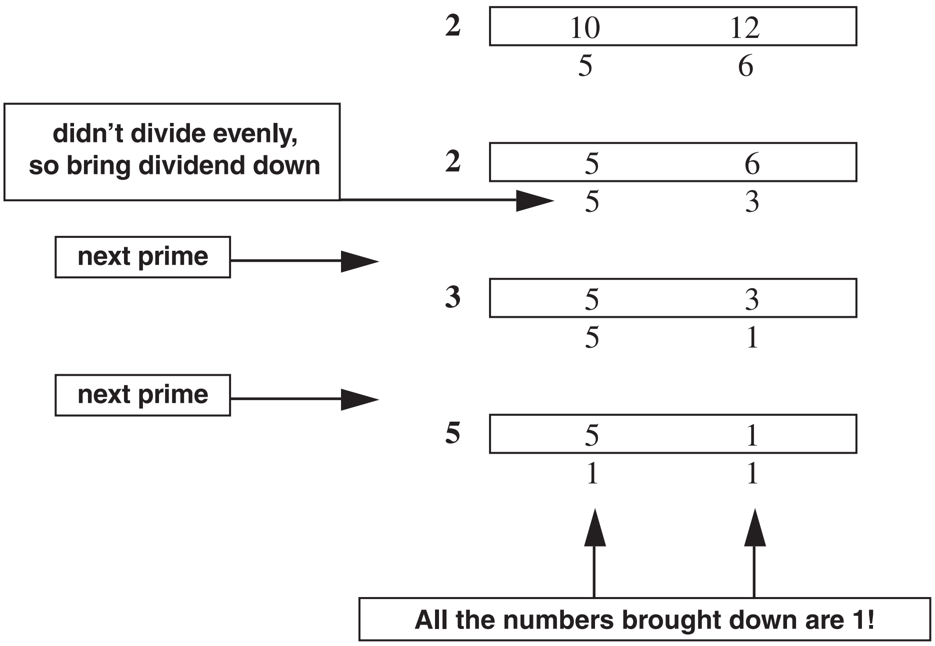


continued on next page

METHOD #3 (continued)

Step	**Action**
4	Repeat **STEP 3** using the numbers you wrote under the box in each prior division as the new dividends, until:

IF …	**AND …**	**THEN …**
all the numbers brought down are 1		go to **STEP 5**.
all the numbers brought down are *not* 1	the divisor will *not* divide evenly into any of them	select the next unused prime that divides evenly and repeat **STEPS 3–4**.



Step	**Action**
5	Multiply all the divisors used for each of the divisions you did. This product is the lowest common denominator. $2 \times 2 \times 3 \times 5 = 60$

METHOD #3 (continued)

Completing the addition

$$\frac{\mathbf{9}\times 6}{\mathbf{10}\times 6}=\frac{54}{60} \quad \text{and} \quad \frac{\mathbf{7}\times 5}{\mathbf{12}\times 5}=\frac{35}{60} \text{ so,}$$

$$\frac{54}{60}+\frac{35}{60}=\frac{89}{60}=1\frac{29}{60}$$

Another example

Find the LCD for these fractions: $\frac{4}{14}, \frac{33}{108}, \frac{12}{15}$

2	14	108	15
	7	54	15
2	7	54	15
	7	27	15
3	7	27	15
	7	9	5
3	7	9	5
	7	3	5
3	7	3	5
	7	1	5
5	7	1	5
	7	1	1
7	7	1	1
	1	1	1

The lowest common denominator is: 2 × 2 × 3 × 3 × 3 × 5 × 7 = 3,780.

PRACTICE

REINFORCEMENT PROBLEMS: FINDING THE LOWEST COMMON DENOMINATOR

Instructions: Find the lowest common denominator for the fractions in each group. Use whatever method you think works best in the situation. If necessary, you can refer to the list of prime numbers on page 70.

Group	Answer	Group	Answer	Group	Answer
1. $\frac{4}{5}+\frac{1}{2}$		7. $\frac{3}{16}+\frac{5}{4}+\frac{1}{10}+\frac{2}{25}$		13. $\frac{1}{2}+\frac{7}{44}+\frac{2}{11}$	
2. $\frac{4}{9}+\frac{2}{3}$		8. $4\frac{4}{72}+\frac{3}{2}+9\frac{5}{18}$		14. $\frac{1}{11}+\frac{8}{3}+\frac{2}{5}$	
3. $\frac{1}{3}+2\frac{1}{7}$		9. $2\frac{1}{6}+1\frac{1}{8}+\frac{7}{12}$		15. $\frac{1}{9}+2\frac{8}{75}+1\frac{13}{15}$	
4. $\frac{1}{5}+\frac{3}{5}$		10. $\frac{14}{15}+\frac{1}{12}$		16. $\frac{7}{10}+\frac{12}{55}+\frac{9}{22}$	
5. $\frac{1}{3}+\frac{2}{5}+\frac{1}{2}$		11. $\frac{3}{16}+2\frac{3}{4}+5\frac{1}{12}$		17. $\frac{7}{7}+\frac{12}{9}+\frac{15}{20}$	
6. $\frac{1}{5}+\frac{1}{80}+\frac{1}{2}$		12. $\frac{2}{9}+\frac{1}{4}+\frac{1}{18}$		18. $\frac{1}{8}+\frac{1}{81}+\frac{1}{18}$	

SOLUTIONS

1. 10
2. 9
3. 21
4. 5
5. 30
6. 80
7. 400
8. 72
9. 24
10. 60
11. 48
12. 36
13. 44
14. 165
15. 225
16. 110
17. 1,260
18. 648

Although you may have used the prime number factoring method for various items, you almost certainly would have wanted to use this method for the last four items. Here is the prime factoring method shown for the last four items:

15.

3	9	75	15
	3	25	5
3	3	25	5
	1	25	5
5	1	25	5
	1	5	1
5	1	5	1
	1	1	1

3 × 3 × 5 × 5 = 225

16.

2	10	55	22
	5	55	11
5	1	55	11
	1	11	11
11	1	11	11
	1	1	1

2 × 5 × 11 = 110

17. (condensing the table a little)

2	7	9	20
2	7	9	10
5	7	9	5
7	7	9	1
9	1	9	1
	1	1	1

2 × 2 × 5 × 7 × 9 = 1,260

18. (condensing the table a little)

2	8	81	18
2	4	81	9
2	2	81	9
9	1	81	9
9	1	9	1
	1	1	1

2 × 2 × 2 × 9 × 9 = 648

ADDING MIXED NUMERALS WITH UNLIKE DENOMINATORS

Rule

When you add mixed numerals that have unlike denominators, you leave the numbers in the same mixed numeral form, but you must rewrite the fractional parts using the lowest common denominator.

Procedure

The table below shows the procedure for adding mixed numerals with fractions that have unlike denominators. The example shows the addition of $2\frac{3}{8}+4\frac{1}{5}$.

Before you begin …

… you must identify the lowest common denominator (LCD) of the fractions that are part of the mixed numerals. Use whichever of the three methods that you find the easiest. In this example, the LCD is 40.

Step	Action	Example
1	Write the mixed numerals one above the other.	$\begin{array}{r} 2\frac{3}{8} \\ +4\frac{1}{5} \\ \hline \end{array}$
2	Using the LCD, convert the fractional parts of the mixed numerals to equivalent fractions.	$\begin{array}{r} 2\frac{15}{40} \\ +4\frac{8}{40} \\ \hline \end{array}$
3	- Add the whole number parts of the mixed numerals. - Add the fractional parts of the mixed numerals.	$\begin{array}{r} 2\frac{15}{40} \\ +4\frac{8}{40} \\ \hline 6\frac{23}{40} \end{array}$
4	If the fractional part is equal to or greater than 1, then increase the whole number and reduce the fraction.	$\frac{23}{40}$ is less than 1, so no adjustment is necessary.

Subtracting Fractions

GENERAL RULE FOR SUBTRACTING FRACTIONS

Rule

To subtract one fraction from another, make sure the denominators are the same numeral. Then subtract the numerators, and write a new fraction with the new numerator over the same numeral for the denominator.

A generalization of this procedure for subtracting fractions can also be expressed by using letters as variables to represent individual numbers:

$$\frac{a}{y} - \frac{x}{y} = \frac{(a-x)}{y}$$

SUBTRACTING FRACTIONS WITH THE SAME DENOMINATOR

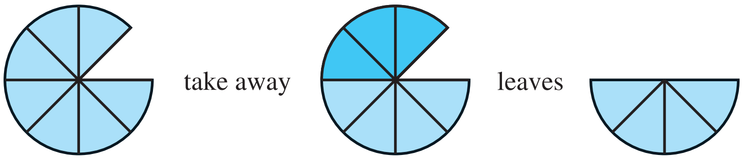
Overview

As you know, the denominator of a fraction shows the number of equal-sized parts in a single whole amount of something. In this discussion, you will see how to subtract fractions that come from whole amounts that are made up of the same number of parts.

Example

Suppose that you want to subtract the fraction $\frac{3}{8}$ from the fraction $\frac{7}{8}$.

In other words: $\frac{7}{8} - \frac{3}{8}$. These fractions are parts of whole amounts that consist of eight total parts. You could visualize the fraction like this, as parts of whole amounts:



$$\frac{7}{8} \qquad - \qquad \frac{3}{8} \qquad = \qquad \frac{4}{8}$$

SUBTRACTING FRACTIONS WITH THE SAME DENOMINATOR (continued)

Procedure

The following table shows the procedure for subtracting fractions *that have the same denominator value*.

Step	Action	Example
1	Subtract the numerators.	Subtract the fractions $\frac{7}{8} - \frac{3}{8}$. Therefore, 7 – 3 = 4.
2	Show the answer as a new fraction with: - the same denominator, and - the numerator is the answer from **STEP 1.**	$\frac{7}{8} - \frac{3}{8} = \frac{4}{8}$
3	If necessary, reduce the fraction to lowest terms.	$\frac{4}{8} = \frac{1}{2}$

SUBTRACTING MIXED NUMERALS WITH THE SAME DENOMINATOR

Procedure

The table on page 78 shows the procedure for subtracting mixed numerals where the fraction parts all *have the same denominator value*.

SUBTRACTING MIXED NUMERALS WITH THE SAME DENOMINATOR (continued)

Procedure (continued)

In the example below, we are subtracting $3\frac{4}{7}$ from $10\frac{1}{7}$.

<table>
<tr><th>Step</th><th>Action</th><th>Example</th></tr>
<tr><td>1</td><td>Vertically align the expressions, placing the larger number above the smaller number.

<i>Note:</i> If any fractional part is greater than 1, convert it to a fraction less than 1, and add the excess to the whole number next to it.</td><td>$\begin{array}{r} 10\frac{1}{7} \\ -3\frac{4}{7} \\ \hline \end{array}$</td></tr>
<tr><td>2</td><td colspan="2">
<table>
<tr><th>IF …</th><th>THEN …</th><th>EXAMPLE</th></tr>
<tr><td>the bottom numerator is smaller than the top numerator,</td><td>go to STEP 3.</td><td></td></tr>
<tr><td>the bottom numerator is bigger than the top numerator,</td><td>borrow 1 from the whole number next to the top numerator and add its equivalent in fractional parts to the top numerator.</td><td>$\cancel{10}\frac{1}{7} = 9\frac{1^{+7}}{7} = 9\frac{8}{7}$

(Because 4/7 is greater than 1/7, 1 is borrowed from the 10. 1 borrowed equals 7/7, which is then added to the numerator.)</td></tr>
</table>
</td></tr>
<tr><td>3</td><td>● Subtract the bottom fraction from the top fraction.
● Subtract the bottom whole number from the remaining top whole number.</td><td>$\begin{array}{r} 9\frac{8}{7} \\ -3\frac{4}{7} \\ \hline 6\frac{4}{7} \end{array}$</td></tr>
<tr><td>4</td><td>If necessary, reduce the fractional part of the answer to lowest terms.</td><td>not necessary in this example</td></tr>
</table>

SUBTRACTING MIXED NUMERALS WITH THE SAME DENOMINATOR (continued)

Example

Flathead Valley Gourmet Shoppe began the week with $45\frac{1}{3}$ pounds of semi-sweet chocolate on hand. At the end of the week, there were exactly $19\frac{2}{3}$ pounds of the chocolate still unused. How much chocolate had been used up?

Step	Example
1	$45\frac{1}{3}$ $-19\frac{2}{3}$ (fractions vertically aligned)
2	Borrowing? Yes—the bottom fraction is bigger than the top fraction. $\cancel{45}\frac{1}{3} = 44\frac{4}{3}$ (3/3 are borrowed from the 45)
3	$44\frac{4}{3}$ $-19\frac{2}{3}$ $25\frac{2}{3}$ (subtract the fractions and whole numbers)
4	Fraction is already in lowest terms. The amount of chocolate used up was $25\frac{2}{3}$ pounds.

SUBTRACTING FRACTIONS AND MIXED NUMERALS WITH UNLIKE DENOMINATORS

Overview

Just like adding fractions and mixed numerals with unlike denominators, it is also necessary to find the lowest common denominator (LCD) when subtracting fractions and mixed numerals. Once you identify the LCD, and convert the unlike fractions to equivalent fractions, then the procedure for subtraction is just the same as you learned in the previous topic.

SUBTRACTING FRACTIONS AND MIXED NUMERALS WITH UNLIKE DENOMINATORS (continued)

Procedure

The table below gives the procedure for subtracting fractions and mixed numerals with unlike denominators.

Step	Action
1	Find the LCD (procedure on page 69) and convert all unlike fractions to equivalent fractions that have the LCD as the denominator.
2	Follow the rules you have learned for subtracting fractions and mixed numerals that have the same denominator.

Example #1

Cecil owned a $\frac{2}{3}$ share in a partnership. However, the partnership was reorganized with new partners, and now Cecil owns a $\frac{3}{8}$ share in the partnership.

a. Did Cecil's share increase or decrease?
b. By what fraction?

Solution #1

a. The LCD for the fractions $\frac{2}{3}$ and $\frac{3}{8}$ is 24. Converting the fractions to equivalent fractions with the same LCD, the resulting fractions are $\frac{2}{3}=\frac{16}{24}$ and $\frac{3}{8}=\frac{9}{24}$. (Now that the fractions have the same denominators, we can see that 9/24 is smaller than 16/24, so we know that Cecil's share decreased.)

b. Subtracting, we get: $\frac{16}{24}-\frac{9}{24}=\frac{7}{24}$. Cecil's share decreased by the fraction $\frac{7}{24}$.

Example #2

The weight of a laptop computer component increased from $12\frac{5}{8}$ ounces to $15\frac{9}{32}$ ounces. How much did the weight increase?

SUBTRACTING FRACTIONS AND MIXED NUMERALS WITH UNLIKE DENOMINATORS (continued)

Solution #2

- The LCD for the fractional part of the mixed numeral is 32. Because the $\frac{9}{32}$ fraction already has the common denominator, we only need to convert the $\frac{5}{8}$ fraction, which results in $\frac{5}{8}=\frac{20}{32}$.

- Subtracting the fractions, we get:

$$\begin{array}{r} 15\frac{9}{32} = \ 14\frac{41}{32} \\ -12\frac{20}{32} \\ \hline 2\frac{21}{32} \end{array}$$

Notice that we had to borrow 32/32 from the 15

The weight increased by the amount of $2\frac{21}{32}$ ounces.

Caution!

Common mistake!

Cancellation *does not work* when subtracting fractions, as it does when multiplying fractions.

When multiplying these fractions, it is OK to do this: $\frac{5}{8}\bullet\frac{4}{9}=\frac{5}{\cancel{4}\cdot 2}\bullet\frac{\cancel{4}}{9}$

but when subtracting the fractions, *cancellation will not give a correct answer*.

$$\frac{5}{8}-\frac{4}{9}=\frac{5}{\cancel{4}\cdot 2}-\frac{\cancel{4}}{9}$$

No!

PRACTICE

REINFORCEMENT PROBLEMS: SUBTRACTING FRACTIONS

Instructions: Subtract and show all fractional answers in lowest terms. Adjust answers that are mixed numerals so that the fractional part is never equal to or greater than 1. Write your answer in the space provided next to each item.

Item	Answer	Item	Answer	Item	Answer
1. $\frac{5}{7} - \frac{2}{7}$		7. $10\frac{1}{2} - 7\frac{1}{6}$		13. $5\frac{6}{7} - 1\frac{9}{10}$	
2. $2\frac{5}{8} - \frac{7}{8}$		8. $115\frac{5}{6} - 90\frac{10}{11}$		14. $24\frac{1}{2} - 11\frac{3}{4}$	
3. $5\frac{7}{12} - 1\frac{3}{12}$		9. $10\frac{5}{16} - 5\frac{1}{4}$		15. $83\frac{1}{9} - 45\frac{1}{3}$	
4. $8 - 4\frac{3}{9}$		10. $\frac{3}{5} - \frac{1}{4}$		16. $\frac{13}{16} - \frac{2}{3}$	
5. $4\frac{3}{8} - 4$		11. $\frac{5}{6} - \frac{1}{2}$		17. $\frac{1}{x} - \frac{1}{y}$	
6. $\frac{21}{10} - \frac{3}{8}$		12. $\frac{1}{2} - \frac{1}{3}$		18. $\frac{a}{x} - \frac{b}{y}$	

SOLUTIONS

(Fractions greater than 1 are converted to mixed numerals.)

1. 3/7
2. 1 3/4
3. 4 1/3
4. 3 2/3
5. 3/8
6. 69/40
7. 3 1/3
8. 24 61/66
9. 5 1/16
10. 7/20
11. 1/3
12. 1/6
13. 3 67/70
14. 12 3/4
15. 37 7/9
16. 7/48
17. $(y - x)/xy$
18. $(ay - bx)/xy$

To double-check your procedure, here are detailed solutions for three of the problems:

3.
(LCD is 12)

$$\begin{array}{r} 5\frac{7}{12} \\ -1\frac{3}{12} \\ \hline 4\frac{4}{12} = 4\frac{1}{23} \end{array}$$

4.
(LCD is 9)

$$\begin{array}{rcr} 8 & = & 7\frac{9}{9} \longleftarrow \boxed{\text{Borrow } \frac{9}{9}} \\ -4\frac{3}{9} & & -4\frac{3}{9} \\ \hline & & 3\frac{6}{9} = 3\frac{2}{3} \end{array}$$

13.
(LCD is 70)

$$\frac{6}{7} = \frac{?}{70} = \frac{6 \times 10}{7 \times 10} = \frac{60}{70}$$

$$\frac{9}{10} = \frac{?}{70} = \frac{9 \times 7}{10 \times 7} = \frac{63}{70}$$

so,

$$\begin{array}{r} 5\frac{60}{70} = 4\frac{130}{70} \longleftarrow \boxed{\text{Borrow } \frac{70}{70}} \\ -1\frac{63}{70} \\ \hline 3\frac{67}{70} \end{array}$$

SIGNED FRACTIONS

Signed numbers explained

The basic operations using signed numbers were presented in the first volume of this series as part of the Essential Math for Accounting section. The operations using signed fractions follow all of the same rules. For a full review of this subject, you should refer to that material in the first volume.

Expressing a signed fraction

A fraction can have three possible signs:

- the sign of the numerator
- the sign of the denominator
- the sign of the entire fraction

When a sign is omitted, it is assumed that an item is positive.

Examples of signs

Suppose we represent fractional amounts by the variables x and y:

- $\frac{x}{y}$ means $+\frac{+x}{+y}$
- $\frac{-x}{y}$ means $+\frac{-x}{+y}$
- $\frac{x}{-y}$ means $+\frac{+x}{-y}$
- $-\frac{x}{y}$ means $-\frac{+x}{+y}$

Writing a negative fraction

Generally, a negative fraction is written with a minus sign next to the fraction, like this: $-\frac{x}{y}$, or a minus sign next to the numerator, like this: $\frac{-x}{y}$.

Fractions are not usually expressed with a minus sign next to the denominator.

Changing value by changing signs

The table below shows you the effect on the value of a fraction when you change its signs.

If you change …	then you …
the sign of *both* the numerator and the denominator,	do *not* change the value of a fraction.
change any two of the three signs of a fraction,	do *not* change its value.
any one of the signs of a fraction,	do change its value.
all three signs of a fraction,	do change its value.

SIGNED FRACTIONS (continued)

Examples

These fractions do *not* change value:

- Both numerator and denominator changed: $\frac{2}{3} = \frac{-2}{-3}$ and $\frac{-2}{3} = \frac{2}{-3}$
- Two signs changed: $-\frac{2}{3} = \frac{-2}{3} = \frac{2}{-3}$

These fractions *do* change value:

- One sign changed: $-\frac{2}{3}$ is the negative of $\frac{2}{3}$ and $\frac{2}{3}$ is the negative of $\frac{-2}{3}$
- All signs changed: $\frac{2}{3}$ is the negative of $-\frac{-2}{-3}$

COMPLEX FRACTIONS

Definition

A complex fraction is a fraction in which either the numerator or denominator or both contains a fraction.

Examples

Here are some examples of complex fractions:

- $\dfrac{\frac{4}{7} \cdot \frac{1}{2}}{10}$
- $\dfrac{\frac{4}{7} - \frac{2}{9}}{\frac{5}{12}}$
- $\dfrac{\frac{5}{8} + \frac{1}{3}}{\frac{4}{5} - \frac{1}{4}}$
- $\dfrac{\frac{x}{y} \cdot \frac{a}{b}}{y}$

All of these look pretty scary! But actually there is an easy way to simplify them.

Rule

To simplify a complex fraction, do the operations in the numerator and the denominator separately. This reduces each one to a single fraction. Then invert the denominator and multiply it by the numerator.

COMPLEX FRACTIONS (continued)

Procedure

The table below demonstrates the procedure for simplifying a complex fraction.

The example used is the fraction $\dfrac{\frac{5}{8}+\frac{1}{3}}{\frac{4}{5}-\frac{1}{4}}$.

Step	Action	Example
1	Reduce the numerator to a single fraction or number.	Add: $\frac{5}{8}+\frac{1}{3}$ Result: $\frac{23}{24}$
2	Reduce the denominator to a single fraction or number.	Subtract: $\frac{4}{5}-\frac{1}{4}$ Result: $\frac{11}{20}$
3	Write the new fraction with single fractions (or numbers) in the numerator and denominator.	$\dfrac{\frac{23}{24}}{\frac{11}{20}}$
4	Invert the denominator and multiply it by the numerator.	$\frac{23}{24}\cdot\frac{20}{11}=$ $\frac{23}{6\cdot\cancel{4}}\cdot\frac{5\cdot\cancel{4}}{11}=\frac{115}{66}$
5	Reduce to lowest terms if necessary.	No further reduction possible. Fraction can be expressed as the mixed numeral $1\frac{49}{66}$.

COMPLEX FRACTIONS (continued)

Another example

Simplify the fraction $\dfrac{\frac{4}{7}\cdot\frac{1}{2}}{10}$.

- Reduce the numerator to one fraction: $\frac{4}{7}\cdot\frac{1}{2}=\frac{4}{14}=\frac{2}{7}$
- Denominator is already one number, so invert it and multiply: $\frac{2}{7}\cdot\frac{1}{10}=\frac{2}{70}$
- Reduce to lowest terms: $\frac{2}{70}=\frac{1}{35}$

▼ *Ratios*

OVERVIEW OF RATIOS

What is a ratio?

A **ratio** is the comparison of one number to another number by showing the two numbers in a fraction.

Purpose of a ratio

A ratio is simply an easy way to compare the size of two numbers, because a ratio clearly shows one number next to another number.

Expressing ratios

Examples:

- If there are eight people in a room, of which five are women and three are men, we could compare the number of men to women by writing $\frac{3}{5}$.

In English, this would be expressed as "a ratio of three to five," which here means three men for every five women.

- We could also compare the number of women to men by writing $\frac{5}{3}$. This would be expressed as "a ratio of five to three."

- The ratio of men to the total number of people is $\frac{5}{8}$.

Also, ratios can be expressed by using a colon, such as 3:5 or 5:3, or 5:8.

OVERVIEW OF RATIOS (continued)

Why use a fraction?

Whenever a ratio is expressed as a fraction, this means that we are also comparing the numbers by division. This is because a fraction is also a way of showing division. Why express a ratio this way? Because it makes it easy to convert the ratio comparison into a **rate**. (Sometimes people prefer to see a number comparison as a rate instead of a ratio.)

A rate is a comparison of numbers by actually doing the division, so that we obtain an amount that shows the quantity of some item to each single unit of another item.

How to get a rate

To calculate a rate, divide the denominator into the numerator. The resulting number shows how many units of the numerator item there are for each single unit of the denominator item.

EXAMPLES OF RATIOS AND RATES

Example #1

"Ashley earned $713 after working for 46 hours." What is the ratio of money earned to hours worked? What is her wage rate?

Ratio	Interpretation	Rate	Interpretation
$\frac{713}{46}$	A ratio of 713 (dollars) to 46 (hours)	15.5	Ashley receives the amount of 15.5 (dollars) for each hourly unit of work (dollars per hour)

Example #2

Jones Company made $24,000 of sales and had $15,000 of gross profit. What is the ratio of sales to gross profit? What is the rate of sales to gross profit?

Ratio	Interpretation	Rate	Interpretation
$\frac{24}{15}$	A ratio of 24 thousand (sales dollars) to 15 thousand (gross profit dollars)	1.6	Jones Company had to make 1.6 dollars of sales for each dollar of gross profit (dollars of sales per dollar of gross profit)

EXAMPLES OF RATIOS AND RATES (continued)

Example #3

For Jones Company, what is the gross profit ratio? What is the gross profit rate?

Ratio	Interpretation	Rate	Interpretation
$\frac{15}{24}$	A ratio of 15 (thousand dollars) to 24 (thousand dollars)	.625	Jones Company earned .625 dollars of gross profit for every dollar of sales (dollars of gross profit per dollar of sales)

Terminology confusion

Sometimes a rate is called a ratio. This is, technically, a misuse of the term, but it has become so common that it is widely accepted. For instance, in example above, you might see the .625 referred to as the "gross profit ratio."

▼ *Averages*

OVERVIEW OF AVERAGES

What is an average?

An **average** is just a special case of using division to find a rate per unit. What makes an average special is that the number you are dividing into (the dividend) is a total that comes from all the individual values in a group of different items.

Purpose of an average

The purpose of an average is to act as a value that is the single most representative number for all the individual values of the items in a group.

Synonym

An average is also called a **mean**.

OVERVIEW OF AVERAGES (continued)

Two common averages

There are two kinds of commonly used averages:

- arithmetic average
- weighted average

THE ARITHMETIC AVERAGE

Definition

An **arithmetic average** is a value that is calculated by dividing the sum of the values of individual items in a group by the number of items in the group, or:

$$\frac{\text{sum of the values}}{\text{number of values}} = \text{Arithmetic Average}$$

When to use

Use an arithmetic average when all of the items in a group are of equal importance or significance—each item has equal weight.

Calculation procedure

The table below shows you how to calculate an arithmetic average.

Step	Action
1	Sum the values of all the individual items in a group.
2	Divide the total from **STEP 1** by the number of items in the group.

THE ARITHMETIC AVERAGE (continued)

Example

Roxanne is taking an accounting class, and there will be three tests of 100 points each in the class. The teacher tells the students that each test will be of equal importance in determining the final grade. At the end of the term, Roxanne has the following test scores: 88, 77, 90. Using these test scores, how should the teacher measure Roxanne's performance in the class?

Because each of the tests are of equal importance, the teacher can calculate an arithmetic average as the number that most fairly represents all of Roxanne's test scores:

Step 1: $88 + 77 + 90 = 255$ (sum of the individual values in the group)

Step 2: $\frac{255}{3} = 85$ (sum of the values divided by the number of values)

Result: 85 is the number that best represents all of Roxanne's test results during the term.

▼ *The Weighted Average*

OVERVIEW

When to use

Use a weighted average when any individual values in a group have different significance or importance. In these circumstances, the weighted average is considered to be the most "fair" average.

Example #1

Lorraine is taking an accounting class. The professor tells the students that there will be three tests: two midterm exams and a cumulative final exam. The midterm exam scores will each be worth 25% of the final grade, and the final exam score will be worth 50% of the final grade. (Note that some of the individual values—the test scores—have different importance.)

OVERVIEW (continued)

Example #2

Overland Park Furniture Store has 10 patio tables in stock. The first three tables cost the store $100 each, and the remaining seven tables cost the store $150 each. (Note that there are a different number of tables at each price.)

Definition of weighted average

A **weighted average** is a number that is calculated by dividing the sum of the weighted values of different items by the sum of the weights, or:

$$\frac{\text{sum of (item value} \times \text{assigned weight)}}{\text{sum of weights}} = \text{Weighted Average}$$

ASSIGN WEIGHTS TO ITEM VALUES

Procedure

The table below shows you how to assign a weight to each item.

Step	Action
1	Identify some measurable factor which shows relative importance or significance of individual items in a group.
2	Calculate the total amount of the factor that applies to the group.
3	Assign a portion of the total in Step 2 to each item in the group, according to each item's importance or significance. *Note:* Be sure that all the portions assigned add up to the total in **STEP 2**.

Example #1, continued

Lorraine is in an accounting class in which there will be two midterm exams each worth 25% of the final grade, and a cumulative final exam that will be worth 50% of the final grade.

Step 1: Here, percent is the measurable factor that shows the relative importance of each exam score.
Step 2: The total amount of the factor for the group is .25 + .25 + .5 = 1 (percent converted to decimals).
Step 3: .25 is assigned as the weight to the score for exam 1, .25 is assigned as the weight for exam 2, and .5 is assigned as the weight for exam 3.

ASSIGN WEIGHTS TO ITEM VALUES (continued)

Example #2, continued

Overland Park Furniture Store has 10 patio tables in stock, and wants to know the average price of all the tables. The first three tables cost $100 each and the other seven tables cost $150 each.

Step 1: The number of tables is the factor that shows the relative importance of each cost value.
Step 2: The total amount of the factor for the group is 3 + 7 = 10.
Step 3: The number 3 is assigned as the weight to the $100 value and the number 7 is assigned as the weight to the $150 value, based on the amount of tables for each value.

HOW TO CALCULATE THE WEIGHTED AVERAGE

Procedure

The table below shows you how to complete a weighted average calculation.

Step	Action
1	Determine the weighted value of each item by multiplying the item value by its assigned weight.
2	Total the weighted values from **STEP 1**.
3	Divide the total of the weighted values from **STEP 2** by the total weighting factor.

Example #1, continued

Step 1: The weighted value of each of Lorraine's test scores are:

- 88 × .25 = 22
- 77 × .25 = 19.25
- 90 × .5 = 45

Step 2: The total of the weighted values is 86.25 (22 + 19.25 + 45)
Step 3: The total of the weights is 1 (.25 + .25 + .5)

Therefore, $\frac{86.25}{1} = 86.25$ is the weighted average score.

HOW TO CALCULATE THE WEIGHTED AVERAGE (continued)

Example #2, continued

For the Overland Park Furniture Store, the steps are:

Step 1: The weighted value of each table value is:

- $100 × 3 = $300
- $150 × 7 = $1,050

Step 2: The total of the weighted values is $1,350 ($300 + $1,050)
Step 3: The total of the weights is 10 (3 + 7)

Therefore, $\frac{\$1,350}{10} = \$135$ is the weighted average cost of the tables

> *Note:* A simple arithmetic average gives the result of $125. However, the weighted average results in a higher amount of $135 because it reflects the greater number of tables at the higher price.

PRACTICE

SOLUTIONS FOR AVERAGES BEGIN ON PAGE 96.

REINFORCEMENT PROBLEMS: AVERAGES

Instructions: For each of the separate situations below:

- Calculate both the arithmetic average and the weighted average.
- When calculating the weighted average, identify the appropriate weighting factor and the total amount of the weighting factor.
- Indicate which average would be most appropriate to use for the situation.

1. Kalamazoo Company inventory records show the information below. The company wants to know the average cost of its inventory.

Item	Units	Cost Per Unit
Beginning balance	100	$.85
Purchase: Feb. 5	1,500	$.98
Purchase: May 22	2,900	$1.15
Purchase: Sept. 9	750	$1.10
Purchase: Nov. 5	1,000	$1.20

2. In Professor Cooper's biology class, there are four exams and one term paper, and each of them count as one-fifth toward the final grade. Greg has earned the following points: exam #1: 82, exam #2: 85, exam #3: 94, exam #4: 96, term paper: 93. What is his average score?

PRACTICE

SOLUTIONS FOR AVERAGES BEGIN ON PAGE 96.

3. Greenberg's Drug Store has eight hourly-rate employees. The table below shows their names and hourly wage rates. What is the average hourly wage?

Hourly Rate	**Employees**
$14.50	Adams, Hoang, Wiglesworth
$13.00	Carreras, Lanahan
$7.50	Barnowski
$5.50	Chiang, Grissom

4. Kirkwood Corporation needs to calculate the average number of shares of stock that were outstanding (owned by shareholders) during last year. This figure will be used in a calculation called "earnings per share." The company has obtained the following information about its outstanding shares during last year:

Date	**Action**	**Shares Outstanding**
January 1	Balance from prior year	100,000
March 1	Sold 7,500 new shares	107,500
May 30	Split the stock 2 for 1	215,000
October 1	Purchased shares from shareholders	210,000

5. During last year, Cynthia enrolled in four credit courses: Biology (4 units), Accounting (4 units), English (3 units), and Speech (3 units). The school uses the following values to determine grade point average (GPA): A = 4, B = 3, C = 2, D = 1, F = 0. Cynthia received the following grades: Biology: A, Accounting: A, English: B, Speech: B. Calculate her GPA.

SOLUTIONS

PRACTICE QUESTIONS FOR AVERAGES BEGIN ON PAGE 94.

REINFORCEMENT PROBLEMS: AVERAGES

1. Arithmetic average: $.85 + $.98 + $1.15 + $1.10 + $1.20 = $5.28   Average is: $\frac{\$5.28}{5} = \$1.056$

   Weighted average: appropriate weighting factor is number of units, which total 6,250.
   The weighted values are: 100 × $.85 = $85, 1,500 × $.98 = $1,470, 2,900 × $1.15 = $3,335, 750 × $1.10 = $825, 1,000 × $1.20 = $1,200, which all total $6,915.

   Average is: $\frac{\$6{,}915}{6{,}250} = \$1.109$

   The weighted average is the most appropriate selection, because there are a different number of chairs at each price.

2. Arithmetic average: 82 + 85 + 94 + 96 + 93 = 450   Average is: $\frac{450}{5} = 90$

   Weighted average: appropriate weighting factor is the fraction 1/5 for each score, which adds to 1.
   The weighted values are: 1/5 × 82 = 16.4, 1/5 × 85 = 17, 1/5 × 94 = 18.8, 1/5 × 96 = 19.2, and 1/5 × 93 = 18.6, which all total to 90.

   Average is: $\frac{90}{1} = 90$

   Notice that the weighted average gives exactly the same result as the arithmetic average, because each item is of equal importance (same weight). So, the arithmetic average, which is easier to calculate and automatically assigns equal importance to each item, is the appropriate choice.

3. Arithmetic average: $14.50 + $13.00 + $7.50 + $5.50 = $40.50.   Average is: $\frac{\$40.50}{4} = \$10.125$

   Weighted average: appropriate weighting factor is number of employees, which total 8.
   The weighted values are: 3 × $14.50 = $43.50, 2 × $13.00 = $26, 1 × $7.50 = $7.50, 2 × $5.50 = $11.00, which all total $88.00.

   Average is: $\frac{\$88}{8} = \$11$

   The weighted average is the most appropriate selection, because there are a different number of employees paid at each wage rate.

4. Arithmetic average: 100,000 + 107,500 + 215,000 + 210,000 = 632,500   Average is: $\frac{632{,}500}{4} = 158{,}125$

   Weighted average: appropriate weighting factor is number of months (you could use weeks or days), which total 12.
   The weighted values are: 2 × 100,000 = 200,000, 3 × 107,500 = 322,500, 4 × 215,000 = 860,000, 3 × 210,000 = 630,000, which all total 2,012,500

   Average is: $\frac{2{,}012{,}500}{12} = 167{,}708$

   The weighted average is the most appropriate method, because each balance of stock is outstanding for a different period of time.

5. Arithmetic average: 4 + 4 + 3 + 3 = 14   Average is: $\frac{14}{4} = 3.5$

   Weighted average: appropriate weighting factor is number of units taken, which is 14. The weighted values are: 4 × 4 = 16, 4 × 4 = 16, 3 × 3 = 9, 3 × 3 = 9, which all total 50.

   Average is: $\frac{50}{14} = 3.57$

   The weighted average is the most appropriate selection, because the units represented by each grade are not all the same.

▼ *Continuation of Basic Algebra Review*

INTRODUCTION

What you should already know

In the first book of this two-volume accounting series, the Essential Math for Accounting section included the following basic concepts with which you should be familiar:

- the order of calculation in a mathematical expression
- calculating with positive and negative numbers
- exponents
- the definition of algebra
- algebra compared to arithmetic
- the concept of isolating a variable

Also, the Essential Math for Accounting section of *this* book showed you how to interpret and calculate with fractions. You should be familiar with that material.

If you are not sure that you fully understand the above topics, it would probably be best for you to review them before starting out here. This continuation of introduction to basic algebra assumes that you understand these topics.

Overview

This Continuation of Basic Algebra Review concentrates on showing you more ways to solve basic equations. You will learn the most important procedures needed to find solutions for basic equations which contain one variable. These are the most common kinds of equations that you will encounter.

▼ *Essential Terminology*

SUMMARY

The essential words and expressions

There are eight very basic words in algebra which you need to understand clearly before you continue this algebra review. A good understanding of these words will make the discussion about solution procedures much easier for you. These are the key words:

- **expression**
- **evaluate**
- **term**
- **simplify**
- **factor**
- **solve**
- **equation**
- **check**

EXPRESSION

Definition

When used in math, the word **expression** is a general word that means any numbers, letters, operational symbols, or grouping symbols, either individually or grouped together.

Examples of expressions

- 5
- $5 + 3$
- $\dfrac{(x + y)}{2}$
- $\dfrac{4}{5} - \dfrac{3}{1} + 8$
- $3x - 12$
- $x^2 + 3$

TERM

Definition

When used in math, the word **term** means any expression or part of expression that is *added or subtracted*.

TERM (continued)

Examples of terms

The table below shows you examples of terms along with examples that are *not* terms.

Expression	Terms	Not Terms
$3 + 7$	$3, 7$	—
$4x - 10y$	$4x, -10y$	$4, x, 10, y$
$5 + x - \frac{3}{8}$	$5, x, \frac{3}{8}$	$3, 8$
$3x + 12x - 2$	$3x, 12x, -2$	$3, x, 12$
$(5 \cdot 8) - 4$	$(5 \cdot 8), -4$	$5, 8$
$2(x - 1) + 4$	$2(x - 1), 4$	$2, (x - 1)$

Coefficient of a term

When a term is a combination of numeral and letter or a numeral and parenthesis, the numeral part of the expression is called a **coefficient**. For example, in the expression $4x$, the 4 is a coefficient. Also, in the expression $5(x + 7)$, 5 is the coefficient of the quantity $(x + 7)$.

Each term has a coefficient

When the coefficient of a term is 1, the 1 is usually not shown. This makes it easy to forget that *every* term has a coefficient, even if it is not shown. For example, in the expression x^2, the coefficient of x is 1. So, $1x^2 = x^2$.

Constant term

When a term is only a numeral, without a letter, that number is referred to as a **constant**. For example, in the expression $3x + 12x - 2$, the -2 is a constant.

Like terms

Like terms are terms that have the same variables and exponents as other terms.

TERM (continued)

Examples of like and unlike terms

The table below shows both like and unlike terms.

Like terms	because …	Unlike terms	because …
$7x, 3x$	the terms have the same variable: x.	$7x, 3y$	the terms have different variables.
$8, -7$	constants are always like terms.	$8, -7x$	one term is a constant and the other is a variable.
$x^2, 4x^2$	the terms have variables with the same exponents.	$x^2, 4x$	the terms have variables with different exponents.
$10xy, 5xy$	the terms have the same variables: xy.	$10xy, 5x$	the terms have different variables.

FACTOR

Definition

When used in math, the word **factor** means any expression or part of expression that is *multiplied*.

Examples of factors

The table below shows expressions that are factors and expressions that are *not* factors.

Factors	because …	Not factors	because …
$7 \cdot 5$	the 7 and 5 are multiplied.	$7 + 5$	the 7 and 5 are added.
$\frac{1}{3} \cdot \frac{9}{4}$	the two expressions are multiplied.	$\frac{1}{3} - \frac{9}{4}$	the two expressions are subtracted.
$5(x + 3)$	the two expressions 5 and $(x + 3)$ are multiplied by each other.	$5 + (x + 3)$	the two expressions 5 and $(x + 3)$ are added to each other.

FACTOR (continued)

Terms with factors

Terms may have factors! In the expression $5y - 10 + 3y^2$, there are three terms ($5y$, 10, and $3y^2$). Notice that the terms $5y$ (5 multiplied by y) and $3y^2$ (3 multiplied by y^2) involve factors—a coefficient and a letter or parenthesis.

EQUATION

Definition

An **equation** is a statement which shows two expressions that are equal to each other.

Examples of equations

- $5 = 5$
- $3x - 6 = 9$
- $12 - 3 = 8 + 1$
- $\left(\frac{7y}{3} + \frac{5}{2y}\right) - 12 = 10\,(y - 2)$

EVALUATE

Definition

When used in math, the word **evaluate** means *to calculate the numerical value* of an expression.

EVALUATE (continued)

Two ways to evaluate

There are only two possible ways to evaluate an expression. The table below shows the alternative possibilities.

If …	Then …	Example: evaluate …
the expression consists entirely of numerical values,	perform the indicated operations. *Note:* You must follow the required order of operation. (See "Mathematical Expressions—How to Evaluate" in Volume 1 Index.)	$4 \cdot 5 + \frac{8}{2}$ Doing the indicated operations, we obtain the result of 24.
the expression includes variables,	● you must have a value to substitute for each variable, and then ● perform the indicated operations.	$4 \cdot 5 + \frac{8}{x}$ Assume that we know the value of x is 4. Then we can evaluate the expression and obtain the result of 22.

SIMPLIFY

Definition

When used in math, the word **simplify** means to reduce an expression to fewer terms or factors, making the expression less complicated. Simplification is done by performing indicated operations and combining terms. These procedures will be explained later on.

> *Note:* Simplifying an expression does not mean that the result is automatically some numerical value. This will only happen when the expression itself contains only numerical values, and has no variables.

SOLVE

Definition

When used in algebra, the word **solve** means to find the value of a variable which makes an equation a true statement.

General approach

As you learned in the first book in this series (and which we will review again), the method for finding the value of the variable is to isolate the variable by itself on one side of the equation.

Examples

- In the equation $x + 8 = 32$, the solution value of x which makes the equation a true statement is 24.
- In the equation $10\frac{x}{2} - 8 = 12$, the solution value of x which makes the equation a true statement is 4.

CHECK

Definition

When used in algebra, the word **check** means to check or verify the proposed solution to an equation. This is done by replacing the variable in an equation with its proposed solution value, and then doing the indicated operations. If the result is a true statement, then the solution is correct.

Examples

- In the equation $x + 8 = 32$, if we replace x with the solution value of 24, and then do the indicated operation by adding 24 plus 8, we get the result: $32 = 32$, which is a true statement. So the solution value must be correct.
- In the equation $10\frac{x}{2} - 8 = 12$, if we replace x with the solution value of 4, and then do the indicated operations by multiplying 10 times the fraction and subtracting 8, we get the result: $12 = 12$, which is a true statement. So the solution value must be correct.

▼ *Equations With a Variable On Only One Side*

OVERVIEW OF PROCEDURES

Introduction

We will begin by studying the steps needed to solve equations that have a variable on only one side. **All of the procedures in this discussion apply only to equations which have a variable that is on only one side of the equation.** Later on, we will study equations that have variables on both sides (see page 127).

Goal: isolate the variable

Our ultimate goal is to be able to solve an algebraic equation. We do this by performing a series of steps which ultimately results in **isolating the variable** on one side of the equation. The steps below accomplish this result.

The steps to follow

If you carefully follow the steps below in sequence, while always keeping an equation in balance, you will soon be solving equations successfully:

- Use the distributive property to remove parentheses.
- Further simplify expressions in the equation by combining like terms.
- Use the addition/subtraction property of equations.
- Use the multiplication/division property of equations.
- Check the solution.

These steps are explained to you in detail below.

STEP #1: USE THE DISTRIBUTIVE PROPERTY

Introduction: the basic idea

The distributive property combines the operations of addition and multiplication. The idea of the distributive property is to show that when you have more than one number to multiply by the same amount, those numbers can either be:

- added together and then multiplied, or
- multiplied separately and then added

and the same answer will result.

STEP #1: USE THE DISTRIBUTIVE PROPERTY (continued)

Example

Consider the expression 3 (8 + 2). How can we calculate an answer to this? The distributive property tells us that we can do either:

- 3 (10) = 30 (add the 8 and 2 in the parenthesis and then multiply), or
- $3 \cdot 8 + 3 \cdot 2 = 24 + 6 = 30$ (multiply the 8 and 2 separately, then add)

General description

The distributive property can be written by using letters as variables to describe any number, as follows: $a (b + c) = ab + ac$

It also works this way: $(b + c)a = ba + ca$

With a variable in the parenthesis

Suppose you have this expression: $4 (x + 3)$

You cannot express the $x + 3$ as a single total. (It is *not* $3x$!) However, using the distributive property, we do know that we can still rewrite this expression as: $4 (x + 3) = 4x + 4(3) = 4x + 12$

And it is also true that: $(x + 3)4 = 4x + 3(4) = 4x + 12$

Any number of terms is OK

The distributive property can be used with any number of terms in a parenthesis. For example: $5(3x + 2x + 4) = 15x + 10x + 20$

Why use the distributive property?

The practical use of the distributive property is that it very nicely removes a parenthesis around terms that are being multiplied by a coefficient. As you will see, this is a major help in simplifying an equation so that it can be solved.

It also works with negative signs

Consider this expression: 3 (7 – 3)
Using the distributive property, we can either do:

- 3 (4) = 12 or,
- $3 \cdot 7 - 3 \cdot 3 = 21 - 9 = 12$

If we have a variable as one of the terms, such as $4 (x - 3)$, then we get:
$4 (x - 3) = 4x - 12$

STEP #1: USE THE DISTRIBUTIVE PROPERTY (continued)

Various combinations of signs

The distributive property can be used to remove a parenthesis regardless of the signs of any of the terms or coefficients involved. There are four possibilities when the coefficient is positive, and there are four possibilities when the coefficient is negative. The table below shows you each situation.

Coefficient Positive	**Coefficient Negative**
$3\,(x + 4) = 3x + 12$	$-3\,(x + 4) = -3x - 12$
$3\,(x - 4) = 3x - 12$	$-3\,(x - 4) = -3x + 12$
$3\,(-x + 4) = -3x + 12$	$-3\,(-x + 4) = 3x - 12$
$3\,(-x - 4) = -3x - 12$	$-3\,(-x - 4) = 3x + 12$

PRACTICE

REINFORCEMENT PROBLEMS: APPLY THE DISTRIBUTIVE PROPERTY

Instructions: In the expressions shown below, remove the parentheses by using the distributive property. Reduce all fractions to lowest terms in your answers.

Expression	Answer	Expression	Answer	Expression	Answer
1. $4(x + 4)$		6. $-2(-c + .75)$		11. $(-4x + 5 + 3)2$	
2. $-8(a + 2)$		7. $(-x - 5).1$		12. $-4(3x - 2y + 5)$	
3. $(y - 10)3$		8. $-8(-2x - 9)$		13. $x(a + b + c)$	
4. $-9(z - 3)$		9. $\frac{4}{3}(3x - 2)$		14. $8\left(\frac{1}{5}x + \frac{3}{8}\right)$	
5. $12(-4x + 5)$		10. $\frac{2}{5}\left(-\frac{3}{12}y + \frac{9}{4}\right)$		15. $\frac{2}{3}\left(-\frac{4}{5}y + \frac{7}{2} - \frac{5}{18}\right)$	

SOLUTIONS

Expression	Answer	Expression	Answer	Expression	Answer
1. $4(x + 4)$	$4x + 16$	6. $-2(-c + .75)$	$2c - 1.5$	11. $(-4x + 5 + 3)2$	$-8x + 16$
2. $-8(a + 2)$	$-8a - 16$	7. $(-x - 5).1$	$-.1x - .5$	12. $-4(3x - 2y + 5)$	$-12x + 8y - 20$
3. $(y - 10)3$	$3y - 30$	8. $-8(-2x - 9)$	$16x + 72$	13. $x\,(a + b + c)$	$xa + xb + xc$
4. $-9(z - 3)$	$-9z + 27$	9. $\frac{4}{3}(3x - 2)$	$4x \quad - \frac{8}{3}$	14. $8\left(\frac{1}{5}x + \frac{3}{8}\right)$	$\frac{8}{5}x + 3$
5. $12(-4x + 5)$	$-48x + 60$	10. $\frac{2}{5}\left(-\frac{3}{12}y + \frac{9}{4}\right)$	$-\frac{1}{10}y + \frac{9}{10}$	15. $\frac{2}{3}\left(-\frac{4}{5}y + \frac{7}{2} - \frac{5}{18}\right)$	$-\frac{8}{15}y + \frac{57}{28}$

STEP #1: USE THE DISTRIBUTIVE PROPERTY (continued)

Overview

After using the distributive property to remove the parentheses that have coefficients other than 1, we can now see how the distributive property applies to expressions in parentheses with a coefficient of 1.

This involves identifying expressions in parentheses that are preceded by either a "+" or a "–."

How to remove parentheses preceded by + or – signs

Although this technique was presented in the first book of this series, it is presented here as part of the complete list of procedures. The table below shows the alternative procedures.

If …	Then …	Example
a plus sign (or no sign) directly precedes the parentheses,	you can eliminate the parentheses and no changes are made to the expression inside it.	$(2x + 1) + (3x - 5) = 10$ can be rewritten as: $2x + 1 + 3x - 5 = 10$
a minus sign precedes the parentheses,	you can eliminate the parentheses, but all signs within the parentheses must be *reversed*.	$-(2x + 3x - 5) = 10$ can be rewritten as: $-2x - 3x + 5 = 10$

Why it works this way

An expression within parentheses that does not show a coefficient has a coefficient of 1, even though the 1 is not written in front of the parentheses. In the first case, removing the parentheses really means that you have simply multiplied everything within the parentheses by +1. So the signs do not change.

In the second case, the signs changed because removing the parentheses really means you have multiplied all terms in the parentheses by the coefficient of –1.

STEP #2: FURTHER SIMPLIFY BY COMBINING LIKE TERMS

Combining like terms

Combining like terms means to combine all the like terms that are on the *same side* of an equation.

How to combine like terms

The table below shows you how to combine like terms. The procedure assumes that you have already completed Step #1, if necessary (using the distributive property to remove parentheses).

Procedure	Action
A	Identify the like terms that are on the same side of the equation.
B	Add or subtract the coefficients of the like terms.
C	Multiply the answer from Procedure B by the variable of the like terms.

Examples

- In the equation $12x - 3x = 36$, combine the like terms.

Procedure A: Identify like terms that are on the same side of the equation: $12x$ and $3x$.
Procedure B: Add or subtract the coefficients of the like terms: $12 - 3$ is 9
Procedure C: Multiply the answer from Procedure B by the variable of the like terms: 9 times x is $9x$

Result: $9x = 36$

STEP #2: FURTHER SIMPLIFY BY COMBINING LIKE TERMS (continued)

Examples (continued)

- In the equation $\frac{38}{5}y+\frac{5}{4}y=128$, combine the like terms.

Procedure A: Identify like terms that are on the same side of the equation: $\frac{38}{5}y$ and $\frac{5}{4}y$ are like terms on the left side. There are no like terms on the right side.

Procedure B: Add or subtract the coefficients of the like terms:

$$\frac{38}{5}+\frac{5}{4}=\frac{152}{20}+\frac{25}{20}=\frac{177}{20}$$

Procedure C: Multiply the answer from Procedure B by the variable of the like terms: $\frac{177}{20}$ times y is $\frac{177}{20}y$

Result: $\frac{177}{20}y=128$

- In the equation $2.5x + 12x – 5x = 20$, combine the like terms.

Procedure A: Identify like terms that are on the same side of the equation: $2.5x$, $12x$ and $5x$ are like terms on the left side.

Procedure B: Add or subtract the coefficients of the like terms:
$2.5 + 12 – 5 = 9.5$

Procedure C: Multiply the answer from Procedure B by the variable of the like terms: 9.5 times x is $9.5x$

Result: $9.5x = 20$

The equation ALWAYS stays equal

Notice that we never did anything that changed the equality of the left side and right side. This is important. The equation must **always** stay equal, no matter what you do to the expressions in the equation.

PRACTICE

REINFORCEMENT PROBLEMS: ELIMINATING PARENTHESES AND COMBINING LIKE TERMS

Instructions: Simplify each of the expressions below by removing parentheses and/or combining like terms as needed.

Expression	**Answer**	**Expression**	**Answer**
1. $3x + 2x$		12. $-(-x + y) + 2x$	
2. $(2y - 3) + 4y - 2$		13. $-x + \frac{3}{11}x - \frac{5}{6}x$	
3. $-(10z - 25) - 8$		14. $a + (-3a - 12b) + 3b$	
4. $2 + (4x - 7) - 3$		15. $4 + (3x - 7) - 8$	
5. $(x + 3) - (5x - 2)$		16. $-\frac{10}{3}x + \frac{1}{4}x - \frac{11}{6}x$	
6. $-(y + 3) + 3y - 10$		17. $(.7y + y) - .2y - .05y$	
7. $-(-a - 1) - (a - 3)$		18. $2x + \frac{x}{3}$	
8. $z + \frac{z}{7} + \frac{1}{3}$		19. $a - \left(1 + \frac{9}{4}a - \frac{4}{5}a\right)$	
9. $(z - 5) - z + 5$		20. $-5 - 2x$	
10. $-(x + 4) + 5x - 10$		21. $(5)(x) - (3)(x)$	
11. $(3x - 5) - (2x - 1)$		22. $-(20e - 12)$	

SOLUTIONS

1. $5x$
2. $6y - 5$
3. $-10z + 17$
4. $4x - 8$
5. $-4x + 5$
6. $2y - 13$
7. 4
8. $\frac{8}{7}z$ + 1/3
9. 0
10. $4x - 14$
11. $x - 4$
12. $3x - y$
13. $-\frac{103}{66}x$
14. $-2a - 9b$
15. $3x - 111$
16. $\frac{-59}{12}x$
17. $1.45y$
18. $\frac{7}{3}x$
19. $\frac{-9}{20}a - 1$
20. $-5 - 2x$
21. $2x$
22. $-20e + 12$

STEP #3: USE THE ADDITION/SUBTRACTION PROPERTY

Definition

The **addition/subtraction property** of equations means that the same quantity can be added to or subtracted from both sides of an equation, and the equation will still remain in balance. That is, the equation's solution will be unaffected.

Example with addition

Consider the equation $3x - 7 = 24$

If we add 7 to both sides of the equation, the equality will be unaffected, and we move closer to isolating the variable x.

add 7 to each side: $3x - 7 \mathbf{+ 7} = 24 \mathbf{+ 7}$
now the equation is: $3x + 0 = 31$
or more simply: $3x = 31$

Example with subtraction

Consider the equation $x + 9 = 5$

If we subtract 9 from both sides of the equation, the equality will be unaffected, and we are able to isolate the variable x.

subtract 9 from each side: $x + 9 \mathbf{- 9} = 5 \mathbf{- 9}$
now the equation is: $x + 0 = -4$
or more simply: $x = -4$

Procedure: use addition/subtraction property

The table on page 113 shows you how to use the addition/subtraction property when you are trying to simplify or solve an equation with a variable on one side.

STEP #3: USE THE ADDITION/SUBTRACTION PROPERTY (continued)

Before you begin … Be sure that you have already combined all like terms.

<table>
<tr><th>Procedure</th><th>Action</th></tr>
<tr><td>A</td><td>Identify which side of the equation the variable is on.</td></tr>
<tr><td>B</td><td>Select a term that appears on the same side of the equation as the variable:
<table>
<tr><th>IF THE TERM …</th><th>THEN</th></tr>
<tr><td>is being added,</td><td><i>subtract</i> that amount from <i>both sides</i> of the equation.</td></tr>
<tr><td>is being subtracted,</td><td><i>add</i> that amount to <i>both sides</i> of the equation.</td></tr>
</table>
</td></tr>
</table>

Example #1: equation is solved

Suppose we have this equation to simplify and solve: $x - 17 = 25$

Procedure A: the variable x is on the left side of the equation
Procedure B: a term that appears on the same side as the variable is –17 add 17 to each side of the equation:

$$x - 17 \mathbf{+ 17} = 25 \mathbf{+ 17}$$
$$x = 42 + 0$$
$$x = 42$$

Result: The solution is 42.

Example #2: equation is solved

Consider the equation: $12.75 = z + 9.83$

Procedure A: the variable z is on the right side of the equation
Procedure B: a term that appears on the same side as the variable is 9.83 subtract 9.83 to each side of the equation:

$$12.75 \mathbf{- 9.83} = z + 9.83 \mathbf{- 9.83}$$
$$2.92 = z + 0$$
$$2.92 = z$$

Result: The solution is 2.92.

STEP #3: USE THE ADDITION/SUBTRACTION PROPERTY (continued)

Example #3: equation is solved

Consider the equation: $4x - 3x + 2 = 24 - 4$

Like terms still need to be combined, which results in: $x + 2 = 20$
Then, apply the addition/subtraction property:

Procedure A: the variable x is on the left side of the equation
Procedure B: a term that appears on the same side as the variable is 2
subtract 2 from each side of the equation:

$$\begin{aligned} x + 2 \mathbf{- 2} &= 20 \mathbf{- 2} \\ x + 0 &= 18 \\ x &= 18 \end{aligned}$$

Result: The solution is 18.

Example #4: equation simplified, but not solved

Suppose we have this equation to simplify and solve: $3x - 11 = 10$

Procedure A: the variable x is on the left side of the equation
Procedure B: a term that appears on the same side as the variable is –11
add 11 to each side of the equation:

$$\begin{aligned} 3x - 11 \mathbf{+ 11} &= 10 \mathbf{+ 11} \\ 3x &= 21 \end{aligned}$$

Result: We have not yet solved the equation (because x is not yet completely isolated), but we have greatly simplified the equation.

Caution

When you are using the addition/subtraction property, be sure that you select a term that is on **same side as the variable**. The following example selects a term on the wrong side. You can see that this does nothing to simplify or solve the equation:

$$\begin{aligned} &x + 8 = 15 \\ &x + 8 - 15 = 15 - 15 \\ &x - 7 = 0 \quad \longleftarrow \boxed{\textbf{Wrong! Not simplified or solved!}} \end{aligned}$$

PRACTICE

REINFORCEMENT PROBLEMS: APPLYING THE ADDITION/SUBTRACTION PROPERTY

Instructions: In the expressions shown below, solve each equation.

Expression	Answer	Expression	Answer
1. $30 = v - 15$		11. $z + 1 = -1$	
2. $-4 = x + 2$		12. $x + 7 = 2$	
3. $z + 5 = -5$		13. $-24 = 74 + x$	
4. $c - 5.75 = -8.44$		14. $\frac{2}{5} + x = \frac{1}{8}$	
5. $10 = x + 11$		15. $-.07 + x = -.095$	
6. $20 = y + 15$		16. $\frac{3}{2} = \frac{1}{4} + x$	
7. $3 = 3 + a$		17. $5 = e - 3$	
8. $-8 + y = 2$		18. $5 + e = 14$	
9. $-15 = 7 + x$		19. $\frac{4}{9} + x = -\frac{2}{3}$	
10. $-2 = -2 + y$		20. $12 = -8 + x$	

SOLUTIONS

1. $45 = v$
2. $-6 = x$
3. $z = -10$
4. $c = -2.69$
5. $-1 = x$
6. $5 = y$
7. $0 = a$
8. $y = 10$
9. $-22 = x$
10. $0 = y$
11. $z = -2$
12. $x = -5$
13. $-98 = x$
14. $-11/40 = x$
15. $x = -.025$
16. $5/4 = x$
17. $8 = e$
18. $e = 9$
19. $x = -10/9$
20. $20 = x$

STEP #4: USE THE MULTIPLICATION/DIVISION PROPERTY

Definition

The multiplication/division property of an equation means that each side of the equation can be multiplied by the same number or divided by the same nonzero number and the equation will still remain in balance. That is, the solution to the equation will be unaffected.

Multiplying an equation

Consider the equation: $\frac{1}{3}x = 21$

The multiplication/division property tells us that we can multiply both sides of this equation by any nonzero number. But what number? Suppose we decide to multiply by 9.

Then: $\mathbf{9} \cdot \frac{1}{3}x = \mathbf{9} \cdot 21$

so, $3x = 189$

Although $\frac{1}{3}x = 21$ and $3x = 189$ are equivalent (the solution is unchanged), we are no closer to actually finding the solution. So, how do we know what number to use for multiplication to get a solution?

Reciprocals

The answer to our problem is the reciprocal. What is a reciprocal?

Two numbers are **reciprocals** when their product is 1. The table below shows some examples of reciprocals.

Number	Reciprocal	Product
5	$\frac{1}{5}$	$5 \cdot \frac{1}{5} = 1$
$\frac{2}{3}$	$\frac{3}{2}$	$\frac{2}{3} \cdot \frac{3}{2} = 1$
–5	$-\frac{1}{5}$	$-5 \cdot \left(-\frac{1}{5}\right) = 1$
$-\frac{8}{7}$	$-\frac{7}{8}$	$\left(-\frac{8}{7}\right) \cdot \left(-\frac{7}{8}\right) = 1$
–1	–1	$-1 \cdot -1 = 1$

STEP #4: USE THE MULTIPLICATION/DIVISION PROPERTY (continued)

Example: multiply with a reciprocal

Consider again the equation: $\frac{1}{3}x = 21$

Suppose we multiply the equation by the reciprocal of 1/3. This would be: $\mathbf{3} \cdot \frac{1}{3}x = \mathbf{3} \cdot 21$

$$x = 63$$

Result: Multiplying by the reciprocal of 1/3 resulted in a coefficient of 1 for x. This eliminated the fraction as a coefficient and solved the equation, because the variable is now isolated.

Using division to solve an equation

Now we have a new equation: $3x = 21$

We can use division to easily solve this kind of equation, when we divide both sides of the equation by 3:

$$\frac{3x}{\mathbf{3}} = \frac{21}{\mathbf{3}} \quad \text{so, } x = 7$$

Procedure: use multiplication/ division property

The table below shows you how to use the multiplication/division property when you are trying to simplify or solve an equation.

Before you begin …

This table assumes that you have **already applied Steps #1 through #3**. So, the equation should already be in the form of $ax = b$, where x is the variable and a and b are constants.

<table>
<tr><th>Procedure</th><th>Action</th></tr>
<tr><td>A</td><td>Identify which side of the equation the variable is on.</td></tr>
<tr><td>B</td><td>Identify the coefficient of the variable.
<table>
<tr><th>IF THE TERM …</th><th>THEN</th></tr>
<tr><td>is a fraction,</td><td>multiply <i>both sides</i> of the equation by the reciprocal of the coefficient.</td></tr>
<tr><td>is not a fraction,</td><td>divide <i>both sides</i> of the equation by the coefficient.</td></tr>
</table>
</td></tr>
</table>

STEP #4: USE THE MULTIPLICATION/DIVISION PROPERTY (continued)

Example #1

Solve the equation: $34 = 5y$

Procedure A: The variable is on the right side of the equation.
Procedure B: The coefficient is a not a fraction, so divide both sides by the coefficient:

$$\frac{34}{\mathbf{5}} = \frac{5y}{\mathbf{5}}$$

Dividing, we get $6.8 = y$

Result: The solution is 6.8.

Example #2

Solve the equation: $-\frac{7}{3}z = 28$

Procedure A: The variable is on the left side of the equation.
Procedure B: The coefficient is a fraction, so multiply both sides by the reciprocal.

Now we can multiply by the reciprocal: $\left(-\frac{\mathbf{3}}{\mathbf{7}}\right)\cdot\left(-\frac{7}{3}\right)z = \left(-\frac{\mathbf{3}}{\mathbf{7}}\right)\cdot 28$

We get $z = -\frac{84}{7}$

which is $z = -12$

Result: The solution is –12.

Example #3

Solve the equation: $5x = \frac{3}{4}$

Procedure A: The variable is on the left side of the equation.
Procedure B: The coefficient is not a fraction, so divide both sides by the coefficient.

$\frac{5x}{\mathbf{5}} = \frac{\frac{3}{4}}{\mathbf{5}}$ which is: $x = \frac{3}{4} \bullet \frac{1}{5}$ which results in: $x = \frac{3}{20}$

Result: The solution is $\frac{3}{20}$.

STEP #4: USE THE MULTIPLICATION/DIVISION PROPERTY (continued)

Example #4

Solve the equation: $\frac{x}{6} = 5\frac{1}{3}$

Procedure A: The variable is on the left side of the equation.
Procedure B: The coefficient is a fraction, so multiply by the reciprocal.

> *Note:* We know the coefficient is a fraction because $\frac{x}{6}$ is the same as $\frac{1}{6}x$.

Multiplying, we get $6 \cdot \frac{x}{6} = 6 \cdot \left(5\frac{1}{3}\right)$

which is $x = \frac{96}{3}$

Result: The solution is $x = 32$.

Example #5

Solve the equation: $-x = 20$

Procedure A: The variable is on the left side of the equation.
Procedure B: $-x$ means $-1x$, so the coefficient of x is -1. The coefficient is a not a fraction, so divide by the coefficient:

$$\frac{-1x}{\mathbf{-1}} = \frac{20}{\mathbf{-1}}$$

which results in $x = -20$.

PRACTICE

REINFORCEMENT PROBLEMS: APPLYING THE MULTIPLICATION/DIVISION PROPERTY

Instructions: Solve each of the equations below. Reduce all fractions to lowest terms in your answers.

Expression	**Answer**	**Expression**	**Answer**
1. $3x = 15$		11. $-4z = -28$	
2. $\frac{1}{3}x = 15$		12. $\frac{3}{7}x = \frac{6}{8}$	
3. $-20 = 5x$		13. $-\frac{4}{9}z = \frac{3}{12}$	
4. $7y = 49$		14. $-x = 5$	
5. $3x = \frac{12}{20}$		15. $.85 = \frac{x}{.25}$	
6. $-30 = -10y$		16. $-y = -3$	
7. $3 = \frac{x}{12}$		17. $5x = \frac{5}{3} + 10$	
8. $20 = -\frac{4}{3}x$		18. $4b = -\frac{3}{8}$	
9. $8r = -12$		19. $5 = -\frac{1}{4}x$	
10. $0 = \frac{x}{2}$		20. $-\frac{2}{10} = \frac{x}{4}$	

SOLUTIONS

1. $x = 5$
2. $x = 45$
3. $-4 = x$
4. $y = 7$
5. $x = 1/5$
6. $3 = y$
7. $36 = x$
8. $-15 = x$
9. $r = -1\ 1/2$
10. $0 = x$
11. $z = 7$
12. $x = 7/4$
13. $z = -9/16$
14. $x = -5$
15. $.2125 = x$
16. $y = 3$
17. $x = 7/3$
18. $b = -3/32$
19. $-20 = x$
20. $-4/5 = x$

MULTIPLICATION/DIVISION: VARIABLE IN THE DENOMINATOR

Situation

You have practiced with equations in which the variable is in the numerator of a fraction. For example in Practice Item #15 on page 120, in the expression $\frac{x}{.25}$ the variable is in the numerator, and you know this is equivalent to $x\frac{1}{.25}$.

However, suppose the variable is the denominator of a fraction? For example, how would you solve the equation $3 = \frac{24}{x}$? How about $3 = \frac{24}{(x+2)}$?

Rule

When there is an equation that contains one fraction with a denominator that is either a single variable, or a quantity in parentheses that contains a single variable, then you can simplify the equation by multiplying all the terms in the equation by the denominator.

Then use whatever steps in the solution procedure that are still needed.

Example #1

Solve the equation: $3 = \frac{24}{x}$

Multiply all terms by the denominator: $3 \cdot \mathbf{x} = \frac{24}{x} \cdot \mathbf{x}$ result: $3x = 24$

Apply multiplication/division property: $3x \div 3 = 24 \div 3$ result: $x = 8$

Example #2

Solve the equation: $3 = \frac{24}{(x+2)}$

Multiply: $3 \cdot \mathbf{(x+2)} = \frac{24}{(x+2)} \cdot \mathbf{(x+2)}$ result: $3(x+2) = 24$

Apply distributive property: $\mathbf{3} \cdot x + \mathbf{3} \cdot 2 = 24$ result: $3x + 6 = 24$

Apply addition/subtraction property: $3x + 6 \mathbf{- 6} = 24 \mathbf{- 6}$ result: $3x = 18$

Apply multiplication/division property: $3x \div \mathbf{3} = 18 \div \mathbf{3}$ result : $x = 6$

More algebra needed …

The subject of variables in the denominators of fractions is a more complex area than shown here. The procedure shown above is a useful tool when you encounter the above common situations. However, the rest of the topic is beyond the scope of this book, but you can study it in a basic algebra textbook.

STEP #5: CHECK THE SOLUTION

Rule: how to check a solution

To check the solution of any equation, substitute the value of the solution for the variable in the original equation. Then evaluate the equation. If a true statement results, the solution is correct.

Example

Solve and check this equation: $5(x - 3) = 25$

To solve: $5x - 15 = 25$ (distributive property applied)
$5x = 40$ (addition/subtraction property applied)
$x = 8$ (multiplication/division property applied)

To check: Substitute 8 for x and evaluate the expressions in the equation.

$5\,(\mathbf{8}) - 15 =?\ 25$
$40 - 15 =?\ 25$
$40 = 40$ This is a true statement. The solution is correct.

REVIEW OF ALL THE STEPS

Overview

You now have all the steps that you need to solve equations that have a single variable on one side of the equation, and that appear in the general form of $ax + b = c$, where x is the variable and a, b, and c are real numbers and a is the only number that cannot be zero.

How to use the steps

The table on page 123 shows you how to apply the steps that you have learned. Follow all the steps in exact sequence, beginning with Step 1.

REVIEW OF ALL THE STEPS (continued)

<table>
<tr><th>Step</th><th>Action</th></tr>
<tr><td>1</td><td>Look for an expression in parentheses.
<table>
<tr><th>IF …</th><th>THEN …</th></tr>
<tr><td>you find an expression in parentheses,</td><td>remove the parentheses by applying the distributive property.</td></tr>
<tr><td>you do not find such an expression,</td><td>go to STEP 2.</td></tr>
</table></td></tr>
<tr><td>2</td><td>Look for like terms on the same side of the equation.
<table>
<tr><th>IF …</th><th>THEN …</th></tr>
<tr><td>you find like terms,</td><td>combine the like terms.</td></tr>
<tr><td>you do not find any of these,</td><td>go to STEP 3.</td></tr>
</table></td></tr>
<tr><td>3</td><td>Look for terms that are added or subtracted on the same side as the variable.
<table>
<tr><th>IF …</th><th>THEN …</th></tr>
<tr><td>you find terms being added or subtracted on the same side as the variable,</td><td>remove them using the addition/ subtraction property.</td></tr>
<tr><td>you do not find any of these,</td><td>go to STEP 4.</td></tr>
</table></td></tr>
<tr><td>4</td><td>Does the variable have a coefficient other than positive 1?
<table>
<tr><th>IF …</th><th>THEN …</th></tr>
<tr><td>the variable has a coefficient other than positive 1,</td><td>change it to 1 by using the multiplication/ division property.</td></tr>
<tr><td>the coefficient is 1,</td><td>go to STEP 5.</td></tr>
</table></td></tr>
<tr><td>5</td><td>Check the solution by substitution.</td></tr>
</table>

REVIEW OF ALL THE STEPS (continued)

Example #1

- Solve: $6x + 2(7x - 10) = 30$

$6x + \mathbf{14x - 20} = 30$ **Step 1:** Distributive property applied
$\mathbf{20x} - 20 = 30$ **Step 2:** Like terms combined
$20x = \mathbf{50}$ **Step 3:** Addition/subtraction property
$x = 2.5$ **Step 4:** Multiplication/division property

$6(\mathbf{2.5}) + 2((7 \cdot \mathbf{2.5}) - 10) =?\ 30$ **Step 5:** Check
$15 + 2(17.5 - 10) =?\ 30$
$15 + 15 =?\ 30$
$30 = 30$ (true)

Example #2

- Solve: $50 = -3(y - 5) - (2y + 25)$

$50 = \mathbf{-3y + 15 - 2y - 25}$ **Step 1:** Distributive property applied
$50 = \mathbf{-5y - 10}$ **Step 2:** Like terms combined
$\mathbf{60} = -5y$ **Step 3:** Addition/subtraction property
$-12 = y$ **Step 4:** Multiplication/division property

$50 =?\ -3(\mathbf{-12} - 5) - (2 \cdot (\mathbf{-12}) + 25)$ **Step 5:** Check
$50 =?\ 36 + 15 - (-24 + 25)$
$50 =?\ 36 + 15 + 24 - 25$
$50 = 50$ (true)

Example #3

- Solve: $(-4c + 3) - (-5c - 2) = 220$

$\mathbf{-4c + 3 + 5c + 2} = 220$ **Step 1:** Distributive property applied
$\mathbf{c + 5} = 220$ **Step 2:** Like terms combined
$c = 215$ **Step 3:** Addition/subtraction property
Step 4: Not needed

$(-4 \cdot \mathbf{215} + 3) - (-5 \cdot \mathbf{215} - 2) =?\ 220$ **Step 5:** Check
$(-860 + 3) - (-1075 - 2) =?\ 220$
$-857 + 1075 + 2 =?\ 220$
$220 = 220$ (true)

REVIEW OF ALL THE STEPS (continued)

Example #4

- Solve: $\frac{4x}{5}+1=19$

$\frac{4x}{5}+1=19$ **Step 1:** Not needed

Step 2: Not needed

$\frac{4x}{5}=\mathbf{18}$ **Step 3:** Addition/subtraction property

$\frac{\mathbf{5}}{\mathbf{4}}\cdot\frac{4}{5}x=\frac{\mathbf{5}}{\mathbf{4}}\cdot 18$ **Step 4:** Multiplication/division property

$x = 22.5$

$\frac{4\cdot\mathbf{22.5}}{5}+1=?19$ **Step 5:** Check

$18 + 1 =? 19$

$19 = 19$ (true)

PRACTICE

REINFORCEMENT PROBLEMS: SOLVING EQUATIONS WITH THE VARIABLE ON ONE SIDE

Instructions: Solve each of the equations below, and check your solutions. Reduce fractions to lowest terms.

Expression	Answer	Expression	Answer
1. $5x + 12 = 27$		13. $-5 = x - 10$	
2. $5y - 3 = 7$		14. $5x = -\frac{4}{9}$	
3. $5z - (2z + 3) = 33$		15. $20 = \frac{4}{a}$	
4. $4(a - 5) = 19$		16. $-5 = \frac{110}{(z - 1)}$	
5. $-10(-x + 12) = 50$		17. $-2(e + 8) - 4 = 5$	
6. $300 = 4(5x - 20)$		18. $4(5 - x) + 2x = 2$	
7. $-5(z - 3) + 22 = 30$		19. $-3 = 5(4a + 1) - 10a$	
8. $-x = -15$		20. $.2(.5x + 8) = 2.4$	
9. $35 = -2(-x + 9) - (x + 1)$		21. $5(x - 3 + 2x) = 12$	
10. $\frac{4}{7}c = 12$		22. $\frac{2}{12}z = 1$	
11. $\frac{1}{3}\left(x - \frac{2}{5}\right) = \frac{7}{8}$		23. $3(b - 10) = 0$	
12. $\frac{x}{3} = 1$		24. $-\frac{1}{2}z = -\frac{1}{8}$	

SOLUTIONS

1. $x = 3$
2. $y = 2$
3. $z = 12$
4. $a = 9.75$
5. $x = 17$
6. $x = 19$
7. $z = 1.4$
8. $x = 15$
9. $x = 52$
10. $c = 21$
11. $x = 3$ 1/40
12. $x = 3$
13. $x = 5$
14. $x = -4/45$
15. $a = 1/5$
16. $z = -21$
17. $e = -12.5$
18. $x = 9$
19. $a = -.8$
20. $x = 8$
21. $x = 1$ 4/5
22. $z = 6$
23. $b = 10$
24. $z = 1/4$

▼ *Equations With a Variable On Both Sides*

OVERVIEW OF PROCEDURE

Our objective

The objective is still the same: to solve the equation by isolating the variable on one side of the equation.

Procedures overview

You will be happy to know that the procedures for solving equations with a **variable on both sides** is practically identical to the procedures you have already learned for solving equations with a variable on one side. The only change is to use addition/subtraction property in Step 3 more than once.

PROCEDURE ILLUSTRATED

Rule

When isolating a variable in an equation that has a variable on both sides, use the addition property twice in Step 3:

- first, to get all terms containing the variable on one side, and
- a second time, to get all other terms on the opposite side of the equation.

Does it matter which side the variable is on?

No, it makes no difference on which side the variable is isolated. The solution will be the same.

Procedure with example

The table on page 128 illustrates the sequence of steps for solving an equation that has a variable on both sides. Focus especially on Step 3. The example is to solve the equation $2(x - 4) = -3(x - 1) - 31$

PROCEDURE ILLUSTRATED (continued)

Step	Action	Example
1	Apply the distributive property.	$2(x - 4) = -3(x - 1) - 31$ becomes $2x - 8 = -3x + 3 - 31$
2	Combine like terms on the same side of the equation.	$2x - 8 = -3x + 3 - 31$ becomes $2x - 8 = -3x - 28$
3	Apply the addition/ subtraction property. You may have to apply the property twice, combining like terms each time you apply it.	● $2x + \mathbf{3x} - 8 = -3x + \mathbf{3x} - 28$ becomes $5x - 8 = -28$ ● $5x - 8 \mathbf{+ 8} = -28 \mathbf{+ 8}$ becomes $5x = -20$
4	Apply the multiplication/ division property.	$5x \div \mathbf{5} = -20 \div \mathbf{5}$ so, $x = -4$
5	Check by substitution.	$2(\mathbf{-4} - 4) =? -3(\mathbf{-4} - 1) - 31$ $-8 - 8 =? 12 + 3 - 31$ $-16 = -16$ (true)

PRACTICE

REINFORCEMENT PROBLEMS: SOLVING EQUATIONS

Instructions: Solve each equation below and check the solution. Express your answer as a fraction if the equation contains fractions, or results in a solution that is a repeating decimal.

Expression	Answer	Expression	Answer
1. $x + 5 = 10 - (x + 1)$		12. $-10(2x + 4) = 5(3x - 2) + (-9x - 8)$	
2. $42 = -6a$		13. $(12)(u) - (8)(u) - 200{,}000 = 0$	
3. $\frac{z}{3} = 10$		14. $100 - 2(c + 2) = 4(5 - 3c)$	
4. $3x - 12 = 0$		15. $.7(x - 3 + 2x) = -2.3(-x - 5)$	
5. $5(y - 1) = 24 - (5y - 11)$		16. $-\frac{x}{2} = -\left(7 + \frac{2}{3}x\right)$	
6. $3(2x - 1) + 2(x + 9) = 87$		17. $2(a - 6) + (a + 1) = a - 11$	
7. $35 = -4(-3 + x) + 5$		18. $-2(-x + 9) = 6x + 4 - 2x$	
8. $\frac{x}{5} = 7 + 2x$		19. $\frac{1}{(x - 3)} = \frac{2}{3}$	
9. $\frac{5}{x} = 7$		20. $-(x + 5) + 2 = 7(x + 4) - 2(x + 1)$	
10. $5(b + \frac{3}{8}) + (b - 2) = 10$		21. $10 - (c - 5) = 2(c + 3) - 6(c + 1)$	
11. $\frac{8}{3} = -\frac{3}{4}(y - 2)$		22. $\frac{1}{3}x = 15000$	

SOLUTIONS

1. $x = 2$
2. $a = -7$
3. $z = 30$
4. $x = 4$
5. $y = 4$
6. $x = 9$
7. $x = -4.5$
8. $x = -3\ 8/9$
9. $x = 5/7$
10. $b = 1\ 11/16$
11. $y = -14/9$
12. $x = -11/13$
13. $u = 50{,}000$
14. $c = -7.6$
15. $x = -68$
16. $x = -42$
17. $a = 0$
18. $x = -11$
19. $x = 4.5$
20. $x = -29/6$
21. $c = -5$
22. $x = 45{,}000$

▼ Formulas

OVERVIEW

Introduction

Formulas are what make algebra really powerful. A formula is the way that algebra can be applied to everyday, real-life problems. There are many different kinds of formulas that can solve all kinds of problems in business, accounting, finance, economics, chemistry, biology, engineering, art, law, and many other applications.

You can think of formulas as mathematical models relating different values.

Definition

A **formula** is an equation that is used to describe a condition or a relationship. A formula is a generalized description, because it uses letters instead of numbers to show all the important elements of the relationship.

Formula for simple interest

A commonly used formula in business and accounting is the formula for the calculation of simple interest. After analyzing all the important elements that relate to the calculation of interest, and how they relate to each other, bankers realized that this was the relationship that always worked:

interest = principal · rate · time

But instead of using words, letters are used instead: $i = prt$

- i is the amount of interest earned
- p is the principal, which means the amount invested or loaned
- r is the percent rate of interest per period, written as a decimal
- t is the number of time periods the money is invested or loaned

Notice that the formula $i = prt$ is really a *generalized* expression that can be used for an interest calculation on *any* investment or loan. It does not use specific numbers, so it does not apply to just one specific investment or loan.

HOW TO USE FORMULAS

Overview

You can use a formula to calculate the solution value for any letter that is in the formula.

Procedure

The table below shows you how to use a formula.

Step	Action
1	Decide which letter in the formula for which you want to find a value. This becomes the variable for which you are solving.
2	From available data, find the numerical value of each of the other letters in the formula for your particular situation.
3	Substitute these numerical values for the letters in the equation.
4	Solve the equation for the variable.

Example #1

Using the formula for interest, suppose that you want to find the amount of interest expense on a two-year loan. The amount of the loan is $15,000 and the bank charges interest at the rate of 9% per year.

Solution: formula is $i = prt$

Step 1: We want to find a value for i.
Step 2: The value for p is $15,000, r is .09, and t is 2.

Step 3: $i = \$15{,}000 \times .09 \times 2$
Step 4: $i = \$15{,}000 \times .09 \times 2$
$i = \$2{,}700$ (for the two-year loan)

Check: $\$2{,}700 = \$15{,}000 \times .09 \times 2$
$\$2{,}700 = \$2{,}700$ (true)

HOW TO USE FORMULAS (continued)

Example #2

Suppose that you invested $15,000 for eight months and you earned $400 interest from the investment. What was the annual rate of interest?

Solution: formula is $i = prt$

Step 1: We want to find a value for r.
Step 2: The value for p is $15,000, i is $400, and t is $\frac{8}{12}$ (8 out of 12 months)

Step 3: $400 = 15{,}000 \times r \times \frac{8}{12}$

Step 4: $400 = 10{,}000\ r$
$.04 = r$ (which is a rate of 4% per year)

Check: $\$400 = (\$15{,}000) \times (.04) \times (\frac{8}{12})$
$\$400 = \$400$ (true)

Changing a formula to be more useful

Frequently, you will encounter a formula that is solved for a particular variable; however, you prefer that the formula would be solved for a different variable, because it would make the formula more useful to you.

Example: The formula $i = prt$ is a formula that is solved for i, the dollar amount of interest on an investment or loan. However, suppose that you frequently need to determine r, the rate of interest. Of course, in each situation, you could always substitute actual values and solve for r, as you did above. But a ready formula just for r would be more efficient. You would like a formula that tells you: $r = ?$

How to change a formula for the variable you need

In order to change a formula so it is solved for the variable you need, do this:

- Identify the variable for which you want to solve.
- Treat all the other items in the formula as constants.
- Solve the formula for the variable you want by isolating the variable in the usual way you learned for equations.

HOW TO USE FORMULAS (continued)

Example: simple interest

Change the formula $i = prt$, so it is solved for r.

- r is the variable for which to solve.
- i, p and t will be treated as constants.
- solving for r:

Steps 1 and 2 do not apply
Step 3: Isolate r: because r is multiplied by both p and t, use the multiplication/division property and divide both sides by pt: $\frac{i}{pt} = \frac{prt}{pt}$

Result: $\frac{i}{pt} = r$

More examples

1. **Accounting Equation Formula:** $a = l + o$ is the formula for the basic financial condition of a business, where a is total value of the assets, l = total liabilities, and o is the owner's claim on assets. Solve the formula for the owner's claim.	$a - l = o$ (addition/subtraction property)
2. **Break-even Formula:** $(s \times u) - (c \times u) - f = 0$ is the formula that determines the break-even units of sales for a business, assuming that the business sells exactly what it produces. s is the sales price per unit, c is the cost per unit, u is the number of units sold, and f is the total fixed costs of the business. Solve the formula for the units of sales needed to break even.	$(s - c)u - f = 0$ (distributive property) $(s - c)u = f$ (addition/subtraction property) $u = \frac{f}{(s - c)}$ (multiplication/division property)
3. **Total Cost Formula:** $T = a + bx$ is the formula that determines total costs of a business, over a particular range of identified cost behavior, where T is the total cost, a is the total fixed cost, b is the cost per unit of product, and x is the number of units produced. Solve the formula for the number of units produced.	$T - a = bx$ (addition/subtraction property) $\frac{T - a}{b} = x$ (multiplication/division property)